EXAMINATION INTO ERROR OF WATTS' DATUM AND RECOMPUTATION OF MOON'S POSITION †

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Abstract

The error in the datum of Watts' lunar charts (Watts,1963) is examined in detail using the lunar occultation observation data obtained in the years 1981 to 1990 and collected and reduced by International Lunar Occultation Centre (ILOC). Based on the result, a table giving the values of the datum error for arbitrary libration arguments l and b and for arbitrary axis angle Π is presented so as to be applied to any positional observation of the Moon. Applying the correction for the error to the determination of the coordinates of the Moon by the lunar occultation observation, the yearly deviations of $\Delta\lambda$ and $\Delta\beta$ are considerably lessened compared with the former analyses to give more accurate observed values for the longitude and latitude of the Moon. Further, fitting a quadratic to the yearly values of $\Delta\lambda$'s, a difference of $-0.35''$/cy in the mean motion of the Moon from the current ephemeris is found, while no significant difference in the acceleration term is detected.
Key words: Lunar occultation, Ephemeris of the Moon.

1. Introduction

When the coordinates of the Moon's center of mass is determined referred to a coordinate system defined by stars, say the FK5 system, the irregularity of the Moon's limb is a stiff obstacle. If the Moon's surface were covered by the sea water to form an equipotential surface as on the Earth, it would be possible, although not easy, to relate the surface with the center of mass. Actually, however, there is no physically significant smooth surface on the Moon. Meanwhile, the datum of Watts' lunar charts (Watts, 1963) provides a substitute for such a surface and it is most commonly used as the reference for the Moon's figure in the reduction of lunar occultation observation and other Moon's observations made for the purpose of determining the coordinates of the Moon.

The lunar limb profile given by Watts' charts for an arbitrary state of libration is constituted by a set of the values for the heights of points on the limb referred to an artificially defined circle, i.e., the zero-level circle. The circle is a cross section of the sphere defining Watts' datum, which is presumed to have a certain radius k with its center at the center of mass of the Moon. We refer to the circle also as Watts' datum or simply as the datum in the following.

It is known, however, that the heights given by Watts' charts have some errors. It is convenient to separate the error for any point on the lunar limb into two parts, the locally averaged value of such errors and the residual. The former should

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be regarded as the error of the datum itself. In other words, we regard the actual Watts' datum as a closed curve which is well approximated by a circle with radius k and center at the center of mass but a little distorted and shifted as a whole due to the effect of this local average of errors. An important fact is that we can determine the local average of the errors with a considerable accuracy by using lunar occultation data while it is not possible to know each residual error. We will analyze the position and shape of the actual datum in the following.

There are many papers on the errors of Watts' charts. Some authors (e.g., Rosselló et al., 1991 and Sôma, 1985) give the deviation of the actual datum from the circle which Watts originally assumed, by finding a correction to the radius or approximating it by an ellipse, as well as calculate the amount of the shift of the center from the center of mass. They obtain those values for either each libration state or the averaged values over all libration states. However, what we need in analyzing the position of the Moon is where any point of Watts' datum is located with respect to the center of mass for arbitrary values of libration arguments l and b. It is another problem to find a circle or an ellipse to fit the limb best or to ask where its center exists, in which we do not have interest in the present study.

Also we must notice to the following fact concerning the relation between the analysis of the datum and the ephemeris of the Moon. It is impossible to obtain an accurate position of the datum relative to the center of mass by observations of the Moon, if a good ephemeris of the Moon is not available. If the coordinates in the ephemeris has a constant error, for example, the position of the datum relative to the center of mass will show a false displacement of the same amount in the opposite direction. It is impossible to know a correct position of the datum. Periodic errors in the ephemeris will be rather more difficult to deal with. Recently, however, we have a very accurate ephemeris of the Moon, such as LE200, whose values for the coordinates of the Moon's center of mass is considered to be almost free from, at least, any periodic error with an amplitude larger than $0.01''$. This makes it possible to compute an accurate position of the datum.

2. Data and Computation Formula

For the purpose of computing the correct position and shape of the datum, it is necessary to analyze observation data of the Moon reduced in a consistent system and in a clearly described formula. In the present study, we use the timing data of lunar occultations in the years from 1981 to 1990 collected and reduced by the International Lunar Occultation Centre (ILOC), where the reduction is made in the following scheme:

(1) The Japanese Ephemeris after 1985 and an ephemeris computed with the same standard before 1984 are used for the coordinates of the Moon's center of mass. The Japanese Ephemeris is virtually the same as LE200, the difference between the tabular values of the Moon's coordinates in the both ephemerides being less than $0.01''$.

(2) The star places are computed based on a star catalogue which is so edited as to be consistent with the FK5 system.

(3) A value of 0.4 second for the personal equation is assigned to all the visual observations.

(4) The coordinates of the telescopes are referred to a world geodetic system which is equivalent to the WGS-84.

(5) The correction for the lunar limb irregularity is made by Watts' charts without any modification, i.e., Watts' datum is regarded as a circle with radius k and center at the center of mass of the Moon.

(6) $k = 0.2725076$ in units of the Earth's equatorial radius is adopted as the radius of the Moon. The value is the one which is recommended by IAU at present.

The number of the timing data used in the analysis below is 98958 in total including 6931 photoelectric data. Those for each year are given in Table 1 together with those of photoelectric data alone. The numbers of the data for each region of libration arguments are also given in Table 2. The process of computing the position

Table 1. Numbers of occultation timing data for every year used in the analysis. Those of photoelectric data are given in the parentheses.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
	8606 (865)	11701 (817)	9830 (968)	8599 (863)	7172 (673)	8212 (536)	10179 (496)	12680 (581)	11161 (642)	10818 (490)

Table 2. Numbers of occultation timing data used in the analysis for every region of libration arguments l and b. Those of photoelectric data are in the parentheses.

$b \setminus l$	-9°	-7°	-5°	-3°	-1°	+1°	+3°	+5°	+7°	+9°
-8°										
	411 (30)	831 (98)	575 (69)	1086 (70)	1366 (91)	1325 (51)	2124 (135)	2916 (157)	1121 (65)	
-6°										
	1062 (93)	1590 (137)	2140 (99)	1279 (100)	1909 (71)	1907 (120)	3798 (157)	4654 (227)	2756 (163)	
-4°										
	727 (54)	1503 (118)	1040 (49)	977 (90)	1040 (70)	1151 (63)	1754 (78)	2599 (187)	592 (76)	
-2°										
	718 (83)	1470 (72)	985 (73)	878 (80)	948 (50)	713 (46)	2043 (92)	2081 (136)	653 (58)	
0°										
	529 (45)	1200 (115)	1632 (117)	1051 (47)	778 (62)	1520 (87)	996 (67)	1823 (140)	606 (46)	
+2°										
	810 (71)	1233 (102)	1207 (77)	892 (68)	776 (69)	1102 (81)	872 (106)	1801 (164)	500 (41)	
+4°										
	996 (104)	2284 (162)	1391 (70)	1905 (68)	1193 (101)	821 (93)	1312 (88)	1779 (120)	394 (29)	
+6°										
	1333 (138)	2957 (213)	1763 (166)	1492 (124)	1287 (115)	1251 (99)	1109 (133)	1317 (154)	333 (41)	
+8°										

and shape of Watts' datum in this study is as follows:

We use $\Delta\sigma$ reduced at ILOC (ILOC, 1993a) as O − C of the distance between the occulted star and the Moon's center of mass at the instant of a phenomenon. In this, O or the observed value is calculated using the lunar coordinates and the star place stated above. On the other hand, C or the computed value is the sum of the Moon's semidiameter corresponding to the radius k and the height obtained from Watts' charts for the point where the phenomenon occurred, both reduced to the values at the Moon's topocentric distance at the time of observation.

We give the position and shape of the datum by a set of values of corrections to the assumed circle of Watts' datum for a number of regions along the limb devided with some interval. We devide the limb circle into 36 regions each with 10° interval of the axis angle Π, the position angle measured from the direction of the Moon's north pole. The correction for each region is computed as the mean of $\Delta\sigma$'s in the region. In the computation, a weight of unity is assigned to all the data except that 0.25 is assigned to the data of grazings.

The position and shape of the datum change depending on the orientation of the Moon, or on the libration arguments l and b. Therefore we must obtain them for various values of l and b.

In calculating the corrections for a given combination of values of l and b, are used all the data for which the argument values of the topocentric libration are within the range of $l\pm2°$ and $b\pm2°$, respectively.

In the above process, all the timing data including visual and photographic observations are used. It is recognized, however, that the effect of the personal equation is not removed perfectly in using such a data set. The effect remains as a whole uniformly in all the regions along the limb, as well as it appears differently depending on the

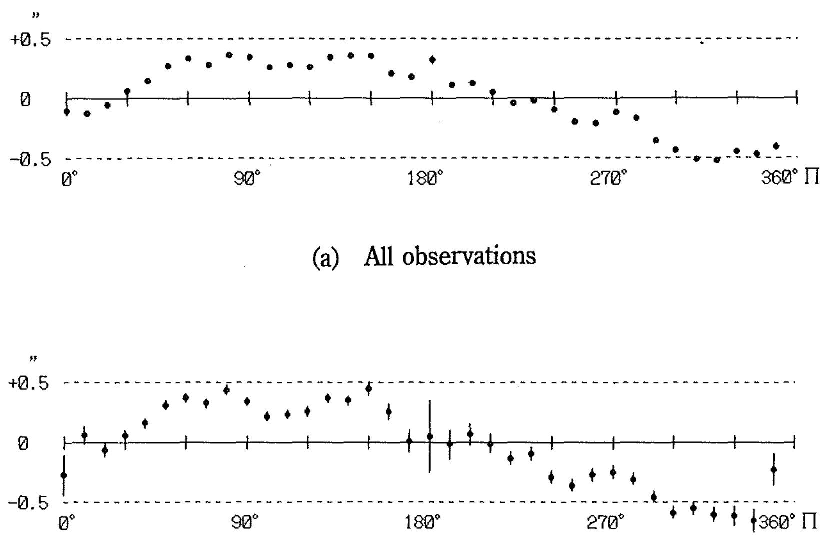


Fig. 1. The error in Watts' datum obtained for two data sets of lunar occultations. The error for the both cases are the averaged one over all states of librations.

axis angle Π. We can see it in Fig. 1. The figures are for two cases, one in which all the data including both visual and photoelectric observations are used (data set (A)) and the other where the photoelectric observations alone are adopted (data set (B)), respectively. The figures are obtained using the data for all values of l and b and therefore show an averaged position and shape of Watts' datum. They should not be used as the reference of the lunar figure in the reduction of the observation of the Moon's position, but they give important information on the systematic correction for the visual data. The comparison of the both figures gives the following systematic difference in the sense of data set (B) minus (A):

$$\Delta h_0 = -0.05'' + 0.08'' \sin \Pi. \qquad (1)$$

This is the same as given in Kubo (1993). When we intend to obtain the datum completely free from the personal equation, we must add this difference further to the correction obtained using the data set (A).

3. Position and Shape of Watts' Datum

Table 3 gives Δh for every 10° of axis angle Π and for the values of l and b each with interval of 2°. Δh is the height above the presumed Watts' circle. Therefore, the correction is to be applied to the limb of the Moon whose center is not offset from the center of mass. The tabulated values have been corrected for the effect of the personal equation in the data set (A) by Equation (1). The values Δh are for the mean distance of the Moon, and therefore, when they are applied, they must be multiplied by the ratio $\sin\pi/\sin\pi_0$, π and π_0 being the horizontal parallaxes of the Moon at the observed time and at the mean distance, respectively.

For an arbitrary set of values of l, b and Π which are between the arguments of the tables, Δh should be calculated by interpolation (or extrapolation for values of l or b outside the respective ranges). It is noticed that a triple interpolation (extrapolation) is necessary. In any case, however, the linear interpolation (extrapolation) is sufficient and a higher order one should not be made. The accuracy of the tabular values is estimated to be about 0.05″. In Fig. 2, curves showing general features of the position and shape of the actual Watts' datum are given for some sets of values of l and b, as an example.

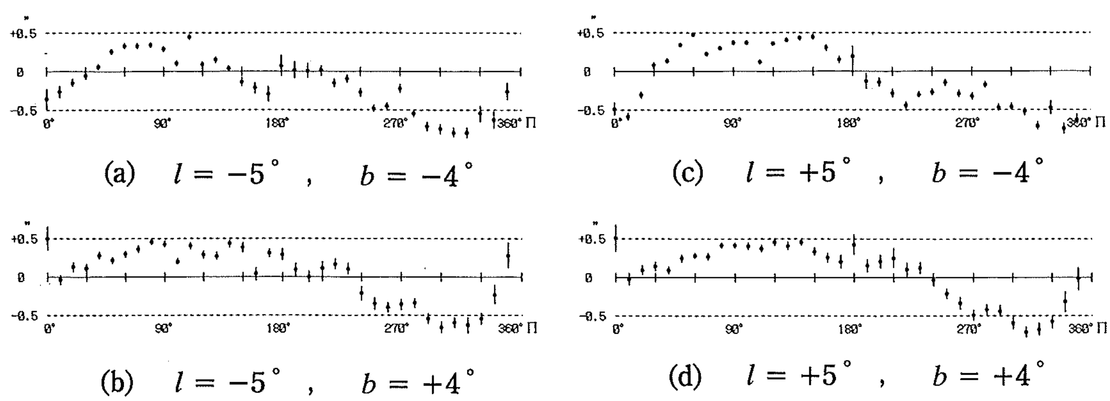


Fig. 2. The error in Watts' datum for some sets of libration arguments l and b.

Table 3 Error of Watts' datum for arbitrary libration argumants l and b and axis angle Π.

Π = 0°

b\\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.60	-0″.52	-0″.27	-0″.36	-0″.58	-0″.59	-0″.42	-0″.23
-4	-0.49	-0.36	-0.40	-0.55	-0.61	-0.67	-0.49	-0.37
-2	-0.34	-0.25	-0.02	-0.47	-0.58	-0.57	-0.47	-0.49
0	-0.16	0.13	0.33	0.15	-0.20	-0.18	-0.28	-0.35
+2	0.41	0.40	0.34	0.58	0.34	0.15	0.09	0.08
+4	0.49	0.50	0.17	0.18	0.75	0.76	0.51	0.31
+6	0.55	0.50	0.28	0.42	0.72	0.43	0.37	0.10

Π = 10°

b\\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.63	-0″.37	-0″.40	-0″.24	-0″.31	-0″.55	-0″.54	-0″.43
-4	-0.33	-0.26	-0.34	-0.31	-0.37	-0.62	-0.58	-0.43
-2	-0.14	-0.13	0.17	-0.24	-0.41	-0.53	-0.37	-0.23
0	-0.03	-0.03	-0.03	-0.20	-0.35	-0.17	-0.08	-0.06
+2	-0.19	-0.01	0.09	0.05	-0.22	-0.15	-0.02	-0.08
+4	-0.12	-0.04	0.15	0.09	-0.05	-0.06	-0.02	0.00
+6	0.19	0.19	0.16	0.07	0.02	0.01	0.06	0.10

Π = 20°

b\\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.17	-0″.25	-0″.23	-0″.11	-0″.17	-0″.29	-0″.32	-0″.34
-4	-0.07	-0.14	-0.21	-0.10	-0.14	-0.28	-0.30	-0.32
-2	-0.04	-0.06	-0.11	-0.10	-0.14	-0.18	-0.24	-0.31
0	0.01	-0.00	-0.06	-0.15	-0.21	-0.12	-0.15	-0.24
+2	-0.07	0.10	0.08	-0.09	-0.30	-0.30	-0.01	0.06
+4	-0.02	0.13	0.12	0.00	-0.19	-0.10	0.10	0.15
+6	0.19	0.20	0.12	0.03	-0.11	-0.14	0.03	0.10

Π = 30°

b\\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.08	-0″.09	-0″.06	0″.07	0″.11	0″.11	0″.03	-0″.02
-4	-0.02	-0.05	-0.10	-0.00	-0.02	0.08	0.08	0.06
-2	0.06	0.02	-0.04	-0.02	-0.00	0.02	-0.01	-0.01
0	0.05	0.01	0.03	-0.01	-0.01	-0.04	-0.08	-0.08
+2	0.16	0.04	0.03	0.07	-0.06	-0.12	-0.04	-0.03
+4	0.27	0.12	-0.06	-0.02	-0.02	0.05	0.14	0.09
+6	0.21	0.18	0.05	-0.07	-0.07	0.05	0.18	0.21

Π = 40°

b\\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.04	-0″.00	0″.01	0″.03	0″.08	0″.23	0″.17	0″.15
-4	0.12	0.06	-0.00	0.06	0.09	0.11	0.14	0.13
-2	0.25	0.23	0.15	0.16	0.16	0.00	0.03	0.10
0	0.27	0.17	0.13	0.14	-0.01	-0.03	0.03	0.10
+2	0.32	0.24	0.18	0.15	-0.04	-0.08	0.07	0.17
+4	0.26	0.28	0.26	0.30	0.22	0.01	0.10	0.15
+6	0.25	0.20	0.23	0.30	0.30	0.12	0.15	0.15

Π = 50°

b\\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.41	0″.28	0″.18	0″.23	0″.40	0″.46	0″.37	0″.30
-4	0.34	0.26	0.16	0.23	0.35	0.40	0.35	0.32
-2	0.30	0.29	0.19	0.23	0.24	0.18	0.25	0.31
0	0.25	0.30	0.27	0.25	0.20	0.15	0.27	0.30
+2	0.20	0.27	0.33	0.36	0.17	0.21	0.23	0.15
+4	0.20	0.22	0.30	0.35	0.25	0.22	0.25	0.22
+6	0.23	0.19	0.24	0.30	0.24	0.26	0.35	0.37

Π = 60°

b\\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.26	0″.30	0″.27	0″.27	0″.43	0″.53	0″.61	0″.68
-4	0.27	0.33	0.32	0.23	0.34	0.42	0.48	0.55
-2	0.34	0.36	0.36	0.31	0.26	0.28	0.37	0.44
0	0.37	0.38	0.37	0.27	0.15	0.20	0.44	0.48
+2	0.39	0.40	0.36	0.21	0.13	0.17	0.33	0.36
+4	0.35	0.30	0.25	0.13	0.17	0.24	0.28	0.27
+6	0.39	0.29	0.23	0.28	0.27	0.23	0.22	0.19

Π = 70°

b\\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.41	0″.34	0″.28	0″.23	0″.35	0″.36	0″.35	0″.32
-4	0.38	0.33	0.30	0.28	0.35	0.26	0.23	0.22
-2	0.38	0.32	0.24	0.39	0.40	0.25	0.17	0.15
0	0.42	0.41	0.34	0.31	0.21	0.23	0.28	0.24
+2	0.49	0.41	0.36	0.20	0.11	0.19	0.28	0.23
+4	0.52	0.37	0.27	0.15	0.17	0.30	0.27	0.20
+6	0.49	0.33	0.27	0.19	0.19	0.30	0.32	0.28

Π = 80°

b\\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.46	0″.39	0″.34	0″.32	0″.39	0″.36	0″.34	0″.37
-4	0.44	0.34	0.30	0.34	0.45	0.37	0.30	0.28
-2	0.50	0.41	0.34	0.43	0.51	0.43	0.24	0.16
0	0.53	0.46	0.42	0.39	0.30	0.38	0.33	0.20
+2	0.59	0.46	0.44	0.36	0.30	0.39	0.36	0.24
+4	0.60	0.46	0.32	0.34	0.44	0.53	0.42	0.33
+6	0.53	0.40	0.30	0.35	0.46	0.52	0.40	0.33

Π = 90°

b\\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.36	0″.26	0″.29	0″.33	0″.40	0″.40	0″.41	0″.46
-4	0.35	0.29	0.29	0.35	0.46	0.40	0.37	0.39
-2	0.41	0.41	0.33	0.35	0.35	0.34	0.37	0.38
0	0.47	0.40	0.36	0.39	0.29	0.28	0.43	0.48
+2	0.44	0.38	0.30	0.38	0.36	0.35	0.41	0.42
+4	0.42	0.43	0.24	0.29	0.46	0.49	0.42	0.34
+6	0.36	0.34	0.21	0.28	0.44	0.46	0.42	0.42

Π = 100°

b\\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.42	0″.14	0″.07	0″.17	0″.32	0″.36	0″.39	0″.44
-4	0.29	0.11	-0.00	0.08	0.27	0.33	0.37	0.42
-2	0.20	0.08	-0.01	0.06	0.18	0.33	0.30	0.26
0	0.31	0.24	0.16	0.09	0.20	0.34	0.38	0.35
+2	0.25	0.20	0.16	0.18	0.31	0.38	0.42	0.35
+4	0.19	0.20	0.20	0.31	0.43	0.44	0.41	0.34
+6	0.18	0.23	0.30	0.38	0.49	0.45	0.36	0.28

Π = 110°

b\\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.47	0″.42	0″.48	0″.40	0″.35	0″.19	0″.11	0″.14
-4	0.38	0.44	0.44	0.40	0.41	0.18	0.13	0.18
-2	0.40	0.53	0.42	0.31	0.27	0.35	0.25	0.17
0	0.57	0.55	0.40	0.21	0.17	0.27	0.24	0.18
+2	0.56	0.47	0.29	0.18	0.22	0.25	0.24	0.23
+4	0.48	0.41	0.28	0.28	0.30	0.38	0.38	0.32
+6	0.33	0.32	0.31	0.26	0.29	0.44	0.38	0.34

Table 3 (continued)

Π = 120°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.05	0″.04	0″.06	0″.25	0″.42	0″.39	0″.37	0″.38
-4	0.05	0.09	0.14	0.18	0.34	0.35	0.36	0.39
-2	0.08	0.17	0.18	0.21	0.29	0.40	0.36	0.33
0	0.23	0.29	0.26	0.35	0.38	0.36	0.28	0.24
+2	0.33	0.36	0.27	0.20	0.27	0.25	0.27	0.27
+4	0.28	0.29	0.26	0.23	0.28	0.42	0.45	0.34
+6	0.20	0.28	0.31	0.30	0.35	0.51	0.44	0.33

Π = 130°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.13	0″.07	0″.15	0″.48	0″.54	0″.50	0″.42	0″.34
-4	0.16	0.15	0.19	0.43	0.49	0.52	0.41	0.31
-2	0.17	0.24	0.32	0.32	0.34	0.54	0.40	0.29
0	0.27	0.33	0.43	0.46	0.39	0.41	0.32	0.30
+2	0.40	0.36	0.39	0.44	0.36	0.39	0.40	0.36
+4	0.30	0.27	0.35	0.38	0.29	0.33	0.41	0.40
+6	0.32	0.35	0.32	0.26	0.39	0.41	0.28	0.24

Π = 140°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.06	-0″.08	0″.02	0″.36	0″.45	0″.40	0″.45	0″.49
-4	0.02	0.04	0.09	0.30	0.37	0.39	0.43	0.47
-2	0.24	0.30	0.27	0.29	0.34	0.41	0.41	0.39
0	0.35	0.38	0.36	0.34	0.37	0.42	0.39	0.35
+2	0.31	0.48	0.51	0.49	0.38	0.43	0.41	0.33
+4	0.33	0.43	0.50	0.51	0.35	0.39	0.46	0.43
+6	0.32	0.39	0.43	0.43	0.44	0.41	0.40	0.47

Π = 150°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.39	-0″.20	-0″.03	0″.10	0″.26	0″.46	0″.49	0″.50
-4	-0.22	-0.14	0.06	0.20	0.36	0.43	0.45	0.52
-2	0.17	0.16	0.22	0.44	0.59	0.51	0.40	0.38
0	0.36	0.40	0.47	0.59	0.59	0.50	0.38	0.39
+2	0.29	0.41	0.55	0.50	0.43	0.44	0.39	0.43
+4	0.30	0.39	0.57	0.53	0.47	0.36	0.33	0.36
+6	0.42	0.48	0.55	0.50	0.37	0.27	0.21	0.20

Π = 160°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.31	-0″.36	-0″.28	-0″.27	-0″.20	0″.05	0″.22	0″.24
-4	-0.25	-0.21	-0.08	0.05	0.17	0.29	0.31	0.30
-2	0.13	0.13	0.15	0.33	0.33	0.37	0.39	0.36
0	0.34	0.48	0.51	0.47	0.39	0.34	0.30	0.27
+2	0.35	0.51	0.55	0.45	0.40	0.37	0.27	0.26
+4	-0.16	0.04	0.38	0.31	0.45	0.26	0.26	0.34
+6	-0.04	-0.04	0.27	0.36	0.34	0.15	0.34	0.47

Π = 170°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.57	-0″.49	-0″.43	-0″.29	-0″.11	-0″.02	-0″.03	-0″.08
-4	-0.49	-0.29	-0.19	-0.06	0.23	0.24	0.15	0.12
-2	-0.09	0.03	0.11	0.08	0.29	0.34	0.27	0.30
0	0.15	0.43	0.47	0.35	0.32	0.43	0.38	0.36
+2	0.43	0.68	0.54	0.37	0.56	0.55	0.42	0.30
+4	0.34	0.31	0.39	0.44	0.43	0.10	0.21	0.35
+6	0.41	0.43	0.41	0.48	0.40	0.04	0.09	0.26

Π = 180°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.11	0″.55	-0″.10	-0″.18	-0″.06	0″.02	0″.16	0″.10
-4	0.00	0.08	0.05	0.05	0.24	0.30	0.20	0.11
-2	0.07	-0.02	0.10	0.30	0.35	0.34	0.39	0.34
0	0.02	0.05	0.30	0.51	0.45	0.34	0.55	0.59
+2	0.37	0.26	0.30	0.50	0.51	0.47	0.57	0.49
+4	0.32	0.29	0.24	0.25	0.58	0.38	0.43	0.34
+6	0.28	0.29	0.21	0.20	0.45	0.36	0.42	0.56

Π = 190°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.17	-0″.10	0″.15	0″.53	0″.30	-0″.23	-0″.32	-0″.35
-4	0.17	0.02	0.02	0.33	0.29	0.01	-0.12	-0.20
-2	0.17	0.19	0.10	0.19	0.35	-0.11	-0.09	0.22
0	0.32	0.28	0.22	0.34	0.46	-0.12	-0.12	0.15
+2	0.30	0.27	0.24	0.12	0.13	0.28	0.33	0.40
+4	0.26	0.09	-0.00	-0.04	0.00	0.12	0.15	0.07
+6	0.45	0.21	0.01	-0.01	0.12	-0.08	-0.04	-0.14

Π = 200°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.12	-0″.05	-0″.16	-0″.02	-0″.13	-0″.16	-0″.13	-0″.04
-4	0.15	0.01	-0.10	0.29	0.21	-0.07	-0.13	-0.05
-2	0.06	0.01	-0.00	0.45	0.47	0.05	0.04	0.01
0	-0.03	-0.09	-0.02	0.23	0.31	0.11	0.07	-0.01
+2	0.12	0.14	0.31	0.29	0.12	0.21	0.22	0.12
+4	0.11	0.00	0.08	0.01	-0.04	0.07	0.21	0.16
+6	-0.00	-0.06	0.00	-0.10	0.09	0.17	0.20	0.10

Π = 210°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	0″.08	0″.06	0″.19	0″.17	0″.01	-0″.24	-0″.42	-0″.35
-4	0.03	0.02	0.17	0.17	-0.08	-0.18	-0.28	-0.29
-2	-0.04	-0.05	0.12	0.22	0.05	0.00	-0.01	-0.02
0	0.04	0.10	0.25	0.29	0.20	0.09	0.10	0.11
+2	0.19	0.24	0.21	0.28	0.15	0.03	0.21	0.21
+4	0.17	0.12	0.10	0.15	-0.01	0.06	0.25	0.34
+6	-0.08	-0.10	0.02	-0.04	0.01	0.19	0.13	0.18

Π = 220°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.09	-0″.13	-0″.10	-0″.00	-0″.10	-0″.28	-0″.43	-0″.42
-4	-0.11	-0.15	-0.12	0.08	-0.02	-0.23	-0.44	-0.46
-2	-0.23	-0.22	-0.04	0.11	-0.05	-0.08	-0.15	-0.33
0	0.04	-0.04	-0.05	0.10	-0.00	0.07	0.01	-0.04
+2	0.30	0.25	0.11	0.08	-0.03	-0.06	-0.04	0.07
+4	0.15	0.17	0.21	0.13	-0.05	0.09	0.10	-0.01
+6	0.06	0.07	0.05	0.05	0.01	0.03	0.08	-0.07

Π = 230°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.13	-0″.08	0″.16	0″.16	0″.02	-0″.32	-0″.41	-0″.45
-4	-0.15	-0.09	-0.01	0.12	0.10	-0.11	-0.30	-0.45
-2	-0.23	-0.12	-0.09	-0.20	0.15	0.10	-0.02	-0.25
0	-0.05	-0.04	-0.11	-0.10	0.06	0.18	0.15	-0.06
+2	0.06	0.06	0.12	0.03	0.00	0.17	0.15	0.09
+4	0.04	0.11	0.21	0.03	-0.08	0.08	0.12	0.09
+6	0.02	0.02	0.00	-0.13	0.04	0.17	0.03	-0.18

Table 3 (continued)

Π = 240°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.31	-0″.27	-0″.11	-0″.12	-0″.13	-0″.22	-0″.42	-0″.40
-4	-0.33	-0.27	-0.15	-0.14	-0.09	-0.07	-0.27	-0.39
-2	-0.52	-0.33	-0.20	-0.05	0.04	-0.01	-0.00	-0.13
0	-0.38	-0.27	-0.14	-0.08	-0.09	-0.08	0.06	0.04
+2	-0.10	-0.09	-0.08	-0.12	-0.15	-0.09	0.02	0.02
+4	-0.17	-0.21	-0.34	-0.35	-0.13	-0.10	-0.04	0.00
+6	-0.17	-0.21	-0.37	-0.53	-0.35	-0.16	-0.04	0.03

Π = 250°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.64	-0″.56	-0″.34	-0″.29	-0″.34	-0″.32	-0″.27	-0″.24
-4	-0.58	-0.48	-0.29	-0.27	-0.24	-0.13	-0.14	-0.20
-2	-0.52	-0.42	-0.40	-0.39	-0.08	-0.12	-0.20	-0.26
0	-0.52	-0.48	-0.43	-0.41	-0.27	-0.16	-0.15	-0.23
+2	-0.38	-0.38	-0.34	-0.40	-0.34	-0.10	-0.10	-0.19
+4	-0.23	-0.35	-0.50	-0.54	-0.44	-0.31	-0.21	-0.22
+6	-0.34	-0.36	-0.46	-0.48	-0.31	-0.38	-0.42	-0.43

Π = 260°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.50	-0″.51	-0″.42	-0″.48	-0″.53	-0″.50	-0″.39	-0″.12
-4	-0.51	-0.45	-0.39	-0.47	-0.41	-0.33	-0.29	-0.03
-2	-0.44	-0.44	-0.43	-0.31	-0.19	-0.31	-0.29	0.00
0	-0.43	-0.50	-0.53	-0.33	-0.28	-0.37	-0.33	-0.10
+2	-0.52	-0.47	-0.41	-0.36	-0.37	-0.34	-0.32	-0.26
+4	-0.49	-0.39	-0.45	-0.66	-0.60	-0.35	-0.33	-0.31
+6	-0.50	-0.50	-0.52	-0.53	-0.53	-0.39	-0.30	-0.29

Π = 270°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.23	-0″.18	-0″.07	-0″.17	-0″.18	-0″.32	-0″.39	-0″.18
-4	-0.30	-0.22	-0.13	-0.20	-0.17	-0.24	-0.33	-0.14
-2	-0.48	-0.35	-0.13	-0.03	-0.19	-0.44	-0.43	-0.16
0	-0.44	-0.37	-0.20	-0.16	-0.29	-0.51	-0.43	-0.23
+2	-0.40	-0.36	-0.27	-0.11	-0.07	-0.27	-0.37	-0.31
+4	-0.50	-0.36	-0.08	0.01	0.02	-0.28	-0.49	-0.39
+6	-0.48	-0.29	-0.09	-0.08	-0.22	-0.45	-0.52	-0.37

Π = 280°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.57	-0″.54	-0″.37	-0″.29	-0″.26	-0″.23	-0″.20	-0″.19
-4	-0.56	-0.54	-0.41	-0.31	-0.26	-0.17	-0.17	-0.18
-2	-0.36	-0.44	-0.37	-0.20	-0.20	-0.17	-0.14	-0.19
0	-0.38	-0.48	-0.41	-0.30	-0.34	-0.18	-0.13	-0.24
+2	-0.50	-0.47	-0.35	-0.36	-0.33	-0.13	-0.21	-0.26
+4	-0.37	-0.34	-0.30	-0.36	-0.29	-0.28	-0.42	-0.38
+6	-0.31	-0.27	-0.26	-0.24	-0.21	-0.38	-0.49	-0.50

Π = 290°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.60	-0″.60	-0″.51	-0″.53	-0″.59	-0″.55	-0″.36	-0″.33
-4	-0.71	-0.71	-0.53	-0.44	-0.59	-0.63	-0.47	-0.37
-2	-0.71	-0.66	-0.43	-0.39	-0.70	-0.61	-0.44	-0.42
0	-0.58	-0.53	-0.49	-0.60	-0.67	-0.40	-0.25	-0.29
+2	-0.59	-0.57	-0.61	-0.74	-0.70	-0.53	-0.40	-0.33
+4	-0.51	-0.54	-0.49	-0.55	-0.61	-0.49	-0.43	-0.32
+6	-0.39	-0.40	-0.42	-0.44	-0.46	-0.47	-0.40	-0.27

Π = 300°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.67	-0″.72	-0″.73	-0″.67	-0″.69	-0″.62	-0″.39	-0″.42
-4	-0.68	-0.74	-0.71	-0.63	-0.64	-0.56	-0.46	-0.46
-2	-0.68	-0.78	-0.76	-0.63	-0.60	-0.52	-0.46	-0.44
0	-0.70	-0.81	-0.86	-0.68	-0.61	-0.55	-0.42	-0.38
+2	-0.74	-0.81	-0.88	-0.64	-0.64	-0.66	-0.49	-0.36
+4	-0.67	-0.65	-0.53	-0.41	-0.66	-0.68	-0.60	-0.42
+6	-0.57	-0.54	-0.39	-0.22	-0.48	-0.61	-0.65	-0.44

Π = 310°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.90	-0″.84	-0″.77	-0″.82	-0″.75	-0″.58	-0″.58	-0″.53
-4	-0.85	-0.79	-0.74	-0.90	-0.79	-0.55	-0.52	-0.53
-2	-0.72	-0.64	-0.55	-0.59	-0.66	-0.56	-0.51	-0.52
0	-0.78	-0.77	-0.63	-0.60	-0.63	-0.53	-0.52	-0.48
+2	-0.72	-0.78	-0.76	-0.59	-0.42	-0.29	-0.59	-0.62
+4	-0.63	-0.60	-0.64	-0.55	-0.36	-0.56	-0.71	-0.79
+6	-0.57	-0.57	-0.66	-0.50	-0.35	-0.59	-0.70	-0.74

Π = 320°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.87	-0″.86	-0″.81	-0″.73	-0″.77	-0″.82	-0″.70	-0″.63
-4	-0.78	-0.79	-0.78	-0.71	-0.77	-0.80	-0.70	-0.62
-2	-0.67	-0.69	-0.75	-0.73	-0.68	-0.59	-0.53	-0.55
0	-0.76	-0.81	-0.83	-0.72	-0.63	-0.50	-0.47	-0.52
+2	-0.87	-0.88	-0.69	-0.56	-0.45	-0.40	-0.50	-0.55
+4	-0.65	-0.63	-0.44	-0.34	-0.41	-0.57	-0.67	-0.76
+6	-0.36	-0.38	-0.32	-0.27	-0.38	-0.62	-0.78	-0.72

Π = 330°

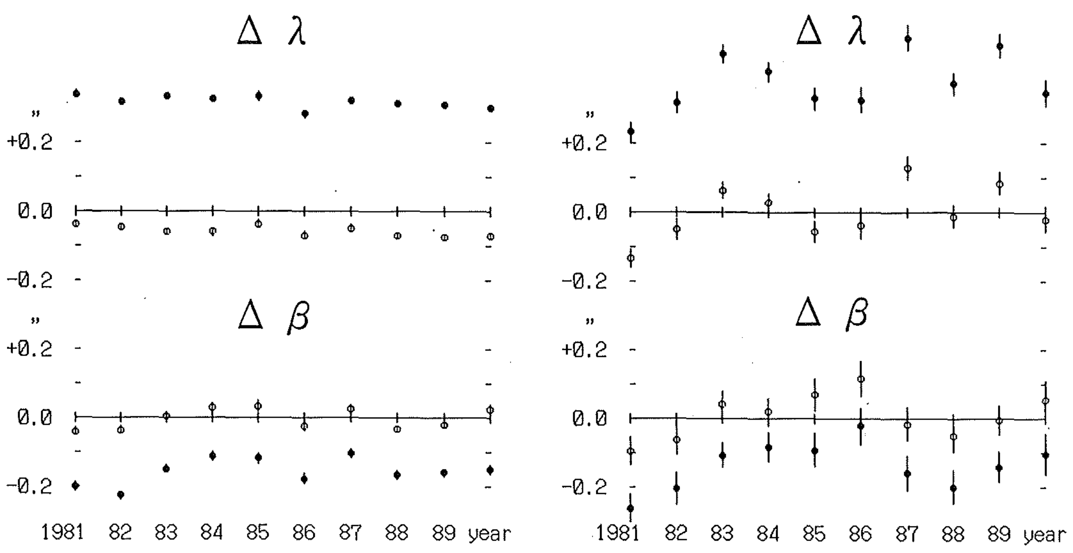
b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.56	-0″.58	-0″.63	-0″.75	-0″.60	-0″.44	-0″.40	-0″.56
-4	-0.55	-0.55	-0.63	-0.76	-0.67	-0.55	-0.47	-0.66
-2	-0.48	-0.54	-0.53	-0.59	-0.75	-0.66	-0.57	-0.70
0	-0.51	-0.60	-0.34	-0.39	-0.58	-0.43	-0.51	-0.56
+2	-0.39	-0.51	-0.40	-0.34	-0.51	-0.44	-0.52	-0.45
+4	-0.37	-0.54	-0.62	-0.60	-0.59	-0.65	-0.58	-0.58
+6	-0.31	-0.40	-0.50	-0.47	-0.44	-0.51	-0.46	-0.49

Π = 340°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.85	-0″.77	-0″.71	-0″.95	-1″.02	-0″.83	-0″.72	-0″.76
-4	-0.66	-0.62	-0.60	-0.96	-1.03	-0.86	-0.74	-0.75
-2	-0.66	-0.71	-0.66	-0.72	-0.95	-0.83	-0.74	-0.59
0	-0.85	-0.79	-0.57	-0.51	-0.40	-0.60	-0.48	-0.29
+2	-0.79	-0.62	-0.46	-0.41	-0.21	-0.50	-0.44	-0.28
+4	-0.38	-0.23	-0.26	-0.11	0.15	-0.13	-0.32	-0.38
+6	0.04	0.05	0.04	0.07	-0.08	-0.21	-0.11	-0.10

Π = 350°

b\l	-7	-5	-3	-1	+1	+3	+5	+7°
-6°	-0″.60	-0″.71	-0″.76	-0″.94	-0″.98	-0″.70	-0″.72	-0″.79
-4	-0.51	-0.26	-0.34	-0.93	-0.91	-0.58	-0.62	-0.67
-2	-0.45	-0.25	-0.27	-0.61	-0.37	-0.36	-0.45	-0.32
0	-0.48	-0.47	-0.31	-0.07	-0.29	0.00	-0.08	-0.06
+2	-0.03	-0.09	0.08	0.20	0.06	0.14	0.19	0.11
+4	0.14	0.28	0.22	0.22	0.21	-0.09	-0.01	-0.11
+6	0.02	0.07	0.21	0.36	0.19	-0.08	-0.03	-0.10



(a) All observations (b) Photoelectric observations

Fig. 3. Corrections for every year to the longitude and latitude of the Moon's center of mass in the current ephemeris. The filled circles and the circles show the values before and after the datum correction, respectively. The values are for the data set of all observations (a) and for that of photoelectric observations alone (b).

4. Lunar Coordinates Corrected for the Datum Error

Although the accuracy of determining the error of the datum is not very high, it produces a considerable improvement to the determination of the lunar coordinates because it removes a systematic error in the analysis.

Fig. 3 gives $\Delta\lambda$ and $\Delta\beta$ of the Moon's center of mass with respect to those tabulated in the ephemeris for the both cases with and without the datum correction, for every year from 1981 to 1990. Fig. 3(a) is for the data set (A) and 3(b) for (B). The weight of 1 is assigned to ordinary (total) occultations and 0.25 to grazings in the least squares solutions. It is noted that this system of weighting is not the same as in the analysis at ILOC (ILOC, 1993b).

First we pay attention to the scatterings of the yearly values of $\Delta\lambda$ and $\Delta\beta$. We can clearly see that it is smaller both in $\Delta\lambda$ and $\Delta\beta$, especially in $\Delta\beta$ for the data set (A), when the datum correction is applied, showing an evident improvement. The absolute values of O - C are not so significant, but it is not accidental that the mean values of $\Delta\lambda$ and $\Delta\beta$ after the correction for the data set (B) are equal to zero. We have constructed the datum correction so that we may have such a result on the assumption that the ephemeris of the Moon gives the correct coordinates of the center of mass. On the other hand, the values before the correction represent $\Delta\lambda$ and $\Delta\beta$ for an approximate center of Watts' datum, which is offset from the center of mass by about $+0.4''$ and $-0.2''$ in longitude and latitude, respectively.

The deviation from zero of the mean values after the datum correction for the data set (A) is due to the the effect of the personal equation. As mentioned in Section 2, the data set (A) has a systematic error given by Equation (1). If we correct

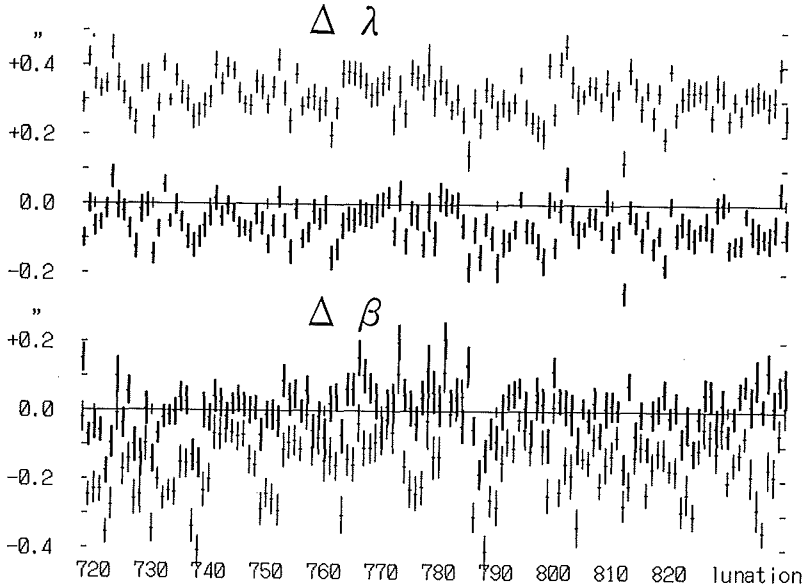


Fig. 4. Corrections for every lunation to the longitude and latitude of the current ephemeris of the Moon. The values with error bar of thin and bold line correspond to before and after the datum correction, respectively. The values are for the data set of all observations.

for Δh_0 of Equation (1) to all $\Delta\sigma$'s for the data set (A) beforehand, then the mean values after the correction in Fig. 3(a) also will reduce to zero. The remaining scattering from year to year is considered to be mostly caused by the inferiority of the star positions.

$\Delta\lambda$ and $\Delta\beta$ for every lunation with the data set (A) are also shown in Fig. 4 for the both cases with and without the datum correction. Here again we can clearly see a sign of improvement due to the correction.

Finally we fit a quadratic with respect to time to $\Delta\lambda$'s for every year and find the coefficients of the linear and quadratic terms. They are respectively,

$-0.35''/\mathrm{cy}$ and $-0.6''/\mathrm{cy}^2$. (epoch : 1986.0)
(± 0.12) (± 4.8) (m.e.)

These values show that the current ephemeris of the Moon seems to have an error of about $-0.4''/\mathrm{cy}$ in the mean motion (thereof in the mean semi-major axis) while no significant error in the acceleration in the ephemeris can be detected in the present study. It is noticed that the above value of the correction to the mean motion is considerably different from Sôma (1985).

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ワッツ月縁図の誤差の解析と月の座標の再計算（要旨）

月の位置観測の整約に参照されるワッツの月縁図の誤差について詳細な解析を行い，任意の秤動引数 l, b 及びワッツ角 Π に対する誤差の値が求められる表を作成した。また，この誤差を補正して，1981～1990年の星食観測により月の重心の座標についての再計算を行うと，年毎のばらつきが相当程度小さくなり，改良のあとがはっきりと示された。さらに，改良された年毎の黄経の値に二次式をフィットさせ，その係数を調べた結果，現在の月の暦との間に $-0.35''$/cy の平均運動の差が認められ，一方，加速度項には差が検出されなかった。