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PERTURBATIONS BY THE FIGURE OF THE MOON IN THE MOTION OF THE MOON

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Abstract

Perturbations by the figure of the Moon in the motion of the Moon are computed semi-analytically and are compared with the solutions formulated by Henrard and Chapront-Touzé. The present solution is different from either of them but is closer to that reached by Chapront-Touzé.

Key words: Lunar theory, figure of the Moon, ephemeris of the Moon

1. Introduction

The effect of the figure of the Moon on the motion of the Moon produces considerably large secular perturbations in the motions of the perigee and the node and small periodic ones in the coordinates of the Moon.

The computation of the perturbations due to the figure of the Moon is one of the most difficult aspects in constructing the ephemeris of the Moon, whether it is made analytically or by numerical integration.

The perturbations have been investigated so far by Henrard (1979) and Chapront-Touzé (1982), but the two solutions are quite different from each other.

In order to determine which solution is correct, the present paper offers another solution for these perturbations, which are computed by means of a semi-analytical method which the present author applied to the computation of other perturbations in the motion of the Moon (Kubo, 1982).

2. Equations of motion

We suppose that the rotational motion of the Moon is given as a function of time, i.e., we assume that the longitudes of the nodes of the Moon's equator coincide with the mean longitudes of those of the lunar orbit, the inclination of the equator has a constant value of 1°32′32″.7 (IAU (1976) system of astronomical constants) and the longitude of the x-axis on the equator is equal to the mean longitude of the Moon plus 180°. In other words we neglect the physical librations of the Moon. This approximation causes some errors in the final result and we estimate them in the next section.

The Hamiltonian of the thus simplified system is

$$F = F_0 + U, \tag{1}$$

where F_0 is the Hamiltonian of the main problem and U is the potential due to the figure of the Moon.

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F_0 is expressed in terms of only the momenta of the Delaunay variables, L, G and H, averaged with respect to the main problem, and does not contain l, g and h. U is a function of all the averaged Delaunay variables and the time t.

The equations of motion are

$$\frac{dL_i}{dt}=-\frac{\partial F}{\partial l_i} \quad \text{and} \quad \frac{dl_i}{dt}=\frac{\partial F}{\partial L_i}, \tag{2}$$

where $L_i = L, G, H$ and $l_i = l, g, h$.

Concerning the potential due to the figure of the Moon, we consider only the terms up to the third-order spherical harmonics and adopt the values for the coefficients of the potential given in Table 5 (IAU (1976) system of astronomical constants).

3. Errors in the solution

Before solving the equations of motion, we will estimate the errors arising from the simplified equations (1) and (2).

As for the errors due to neglecting the physical librations of the Moon, a very rough analysis gives the following features concerning the periodic perturbations in the orbital longitude;

(i) In case the Moon is forced to rotate as described in the previous section, a periodic term $\alpha\cos\omega t$ in the orbital longitude produces a perturbation

$$-\frac{\omega_1^2}{\omega_1^2+\omega^2}\alpha\cos\omega t$$

in the orbital longitude itself due to the coupling between the oribtal and rotational motions through C_{22}, where $\omega_1 \cong 584$ years.

(ii) In case the orientation of the Moon is left free to make physical librations, a periodic term $\alpha\cos\omega t$ in the orbital longitude produces a perturbation

$$\left(\frac{\omega_1}{\omega_0}\right)^2\frac{\omega_0^2}{\omega_0^2-\omega^2}\alpha\cos\omega t$$

in the orbital longitude, where $\omega_0 \cong 2.88$ years.

The difference between the two cases is prominent for $\omega \cong \omega_0$. Especially we have to consider the terms with argument l' and $2g(=2F-2l)$ in the orbital longitude. The effect of assuming the case (i) in place of the case (ii) corresponding to these terms are approximately

$$+0''.0003\sin l' \quad \text{and} \quad +0''.0005\sin 2g \tag{3}$$

in the longitude, respectively.

Also, besides the errors due to the above cause, we need to consider long periodic terms which do not exist in the solution of the main problem and, therefore, are not taken into consideration in the equations of motion in the previous section.

For example, we can not neglect the term $7''.063\sin(\zeta-F)$ in the longitude, which produces a perturbation

$$+0''.00018\sin(\zeta-F) \tag{4}$$

in the longitude, where $\zeta-F=h$.

The terms (3) and (4) are not included in Kubo's solution in Table 1 but should be added to the perturbations in the longitude.

4. Solution

We solve the equatiohs of motion (1) and (2) by the same method which the author applied to other perturbations in the motion of the Moon (Kubo, 1982). The result is shown in Tables 1 to 4 together with those obtained by Henrard (1979) and Chapront-Touzé (1982). Only the terms greater than 0.″00002 for the longitude and latitude and 0.″0000003 for the sine parallax are listed.

The indirect effects are included except in $\dot{g}+\dot{h}$ and $\dot{h}$ by Chapront-Touzé and those by Kubo corresponding to them in Table 4. The coefficients for the gravitational potential of the Moon are taken from IAU (1976) system of astronomical constants in the solutions by the present author and Henrard but Chapront-Touzé adopts a little different values shown in Table 5.

The result of the present computation is different from either of the results by Henrard and Chapront-Touzé but is closer to the lattar.

References

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月の運動における月の形状による摂動（要旨）

久　保　良　雄

月の運動における摂動のうち，月の形状によるものは，効果は小さいが解くことの最もむずかしいものの1つである。数値積分で解く場合，月の軸の動き（秤動）を同時に解くか，月の秤動を既知として解析解を与えるかしなければならない。解析的に解く場合も秤動の正確な解を必要とする。ところが，月の秤動も解くのが困難であり，現在十分な解がない．月の形状による摂動の解析解はこれまでHenrard（1979）とChapront-Touzé(1982)によって求められているが，両者の解は著しく異っている。そのため，筆者は本稿において同じ計算を行ったところ，後者に近い解を得た．まだ多少残っている差異は秤動の扱い方の差によるものと思われる。しかし，その差は黄経で0.″001，地心距離で10cm以下であり，改訂される天体位置表の精度，及び観測の精度に比べて十分に小さい。

Table 1 Perturbations in longitude

Argument				Coefficient of sine		
l	l'	F	D	Kubo	Henrard	Chapront-Touzé
				″	″	″
0	1	0	0	+0.00194	−0.00003	+0.00224
2	0	0	−2	+ 22		+ 25
1	0	0	−2	+ 13	+ 16	+ 14
2	0	−2	0	− 11		+ 43
1	1	0	0	+ 11		+ 14
1	−1	0	0	− 11		− 14
1	0	0	−1	− 7		− 9
1	0	−2	0	− 6		
0	0	2	−2	+ 3		+ 3
1	1	0	−2	+ 2		+ 4
1	−1	0	−2	− 2		− 2
0	1	0	−2	+ 2	− 3	+ 3
0	1	0	2	+ 2		+ 2
3	0	0	0		− 24	
1	0	0	2		− 5	
3	0	0	−2		+ 5	
4	0	0	0		− 4	
2	0	0	2		− 4	
2	1	0	0		+ 2	
2	−1	0	0		− 2	
2	1	0	−2		+ 2	
3	0	0	−4		+ 2	
1	0	0	−4		− 2	
2	0	0	0		+ 2	
1	0	−1	0*			+ 16
0	0	1	0**			+ 4
1	−1	0	−1			− 3
3	0	−2	0			+ 2
0	0	0	2			− 2
2	−1	0	−2			+ 2

* Phase angle is 260° ** Phase angle is 305°

Table 2 Perturbations in latitude

Argument				Coefficient of sine		
l	l'	F	D	Kubo	Henrard	Capront-Touzé
				″	″	″
0	1	1	0	+0.00009		+0.00010
0	1	−1	0	+ 9		+ 10
0	0	1	−2	+ 5	+0.00005	+ 5
2	0	−1	0	− 2	− 2	
1	0	−1	0	+ 2	+ 2	+ 3
2	0	−3	0			+ 2

Table 3 Perturbations in sine parallax

Argument				Coefficient of cosine		
l	l'	F	D	Kubo	Henrard	Chapront-Touzé
				″	″	″
0	0	0	0	−0.0000113	−0.0000120	−0.0000117
0	1	0	0	+ 16		+ 20
1	0	0	0	− 15	− 30	− 4
1	0	0	−2	− 13	− 10	− 12
1	1	0	0	+ 11		+ 11
1	−1	0	0	− 10		− 12
1	0	−2	0	+ 5		
2	0	0	−2	+ 5		+ 4
0	0	0	2	− 4		
0	1	0	−2	− 3		
2	0	0	0		− 10	

Table 4 Secular perturbations in the motions of the perigee and the node

	Kubo	Henrard	Chapront-Touzé
	″/cy	″/cy	″/cy
$\dot{g}+\dot{h}$	− 1.713	− 1.728	
$\dot{h}$	−16.996	−16.983	
(including the indirect effect)			
$\dot{g}+\dot{h}$	− 2.228		− 2.270
$\dot{h}$	−16.862		−16.943
(not including the indirect effect)			

Table 5 Coefficients of the gravitational potential of the Moon

	IAU (1976)	In Chapront-Touzé's solution
C_{20}	−0.0002027	−0.0000203822
C_{22}	+0.0000223	+0.0000022396
C_{30}	−0.000006	−0.000001044
C_{31}	+0.000029	+0.000002860
S_{31}	+0.000004	+0.000000880
C_{32}	+0.0000048	+0.000000482
S_{32}	+0.0000017	+0.000000171
C_{33}	+0.0000018	+0.000000270
S_{33}	−0.000001	−0.000000114