STUDIES ON ABNORMAL REFRACTION OVER SEA SURFACE, I. OBSERVATION OF DIP OF SEA HORIZON

Akira M. Sinzi and Naoaki Owaki

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Abstract

As one of the studies on abnormal refraction over sea surface, made in Sirahama in February, 1959, we have also observed dip of sea horizon under favorable conditions for arising mirage. It is found that the classical linear empirical formula still holds between T_h-T_w, h and dip, where T_h, T_w and h mean the temperatures at the observer and of surface water, and the observer's height from the sea surface, respectively. The coefficients in the formula are obtained to take rather larger values than usually adopted.

Dependence of dip on water vapor is examined and it is regarded to affect dip.

It is also found that there exists a strong correlation between T_h-T_o, and T_h-T_w, where T_o is the temperature in the air immediately above the sea surface. A new formula for dip is suggested.

1. Introduction.

In February, 1959, the authors observed refraction of light over sea surface. Especially we were engaged in observing abnormal refraction causing mirage of distant objects like rocks. The main purposes of our study on refraction at sea are to find out in what conditions abnormal refraction will take place, and to investigate possibility of studying thermal structure in the atmosphere near sea surface through observing refraction.

For the dip of horizon, the following questions are particulary considered:

(1) How closely dip can be approximated by the classical formulae, e. g., by Koss(1899, 1900), Kohlschütter(1903, 1909), and Akiyosi(1930, 1933, 1936), under various conditions in the lowest parts of the atmosphere.

(2) What relationship would actually exist between the temperature T_o, introduced by H. C. Freiesleben(1948), the temperature of the air immediately above the sea surface, and T_h and T_w, the temperatures at the height of the observer and of the water, respectively; though there should be no definite, simple relationship between T_o, T_h and T_w, unless the temperature field in the layers considered is in a stationary state.

The classical formula,

$$\delta=a\sqrt{h}+b(T_h-T_w), \qquad (1)$$

is practically very convenient and useful, because it involves only easily observable quantities, but has not obvious physical meaning. On the contrary, the Freiesleben's formula,

$$\delta=5.04\sqrt{0.1123\,h+T_o-T_h}, \qquad (2)$$

is based on a clear physical consideration, and contains more reasonable parameter T_o, instead of T_w. But it is almost impossible to obtain T_o for practical navigators, or very difficult even for *ad hoc* observers. Hence, it is desired to obtain characters of T_o, and to relate it with the other observable quantities.

Thus, for practical purpose of navigation, one should be contended with the eq. (1), until the characters would be clarified. But the eq. (2) seems to have another use: T_o might be obtained by observing dip, and it might make possible to study heat exchange between the air and the water.

(3) How water vapor will affect the dip. Generally, water vapor in the air has less effect on dip than temperature, but it is sometimes expected that it may play a significant role.

In this paper we shall discuss only on dip based on the results obtained in the entire observation in that session.

2. Results of Observations.

The entire observations were made at Sirahama (135° 59′ E, 34° 43′ N), southeast coast of Izu Peninsula, on the 5th, 6th, 10th, 11th and 12th of February, 1959.

a) For zenith distance observations of horizon and the other distant objects, a Wild T2 theodolite was employed. The instrument was set on a jetty projecting from an islet into a shoal, which lies about 100 meters off the beach, since this station was believed to be least suffered from thermal influence by the land. In the Table 1 are shown the observed values of the zenith distances of the sea horizon by setting the reading knob of the theodolite at left and right positions, respectively, with the times of their observations.

We aimed at a part of horizon having clear image, which was seen through the rocks, whose zenith distances were also observed. These rocks lie 3 miles away from the observing spot within an angular range of about 3° when seen at the theodolite, and scatter in an area 500 metres wide in the east of a head.

Images of the horizon in the telescopic field were largely suffered from scintillation; its periods and amplitudes were averagely of the orders of magnitudes, $0.^s1$—$0.^s2$, and $2''$—$3''$, respectively. These orders of magnitudes changed considerably on some days. Though the accuracy of the theodolite itself is sufficient to the present purpose, the results have considerable errors, which are believed to come about due to scintillation. Generally, on warm, windy weathers scintillation was seemed to be greater. Therefore, mean errors of individual dip observation are approximately $\pm 3''$—$4''$.

b) The vertical distribution of the atmospheric temperatures was observed over a range of height 0 to 4 meters above the sea surface by attaching ten thermometers to a vertically constructed pole in the shoal at about 20 metres

TABLE 1. OBSERVED ZENITH DISTANCE OF SEA HORIZON

J S T		(L)			J S T		(R)		
h	m	°	′	″	h	m	°	′	″
Feb. 5									
7	25	90	4	8	7	35	269	56	7
7	50		4	0	8	0		56	9
8	38		4	13	8	48		55	58
9	30		4	3	9	39		55	49
10	9		4	7	10	15		55	58
10	41		4	8	10	49		56	10
11	17		4	10	11	33		56	12
11	54		3	47	12	0		56	9
12	15		4	3	12	26		56	1
12	45		3	56	12	50		55	51
13	13		3	51	13	20		56	7
13	15		4	3	13	42	269	56	20
Feb. 6									
6	16	90	4	6	6	24	269	56	21
6	31		4	13	6	43		56	14
6	58		3	53	7	10		56	19
8	10		4	7	8	22		56	20
8	46		3	54	9	2		55	57
9	24		3	59	9	30		56	13
10	1		3	45	10	9		56	28
10	22		3	56	10	34		56	23
10	51		3	46	11	0		56	44
11	21		3	37	11	28		56	34
11	38		3	27	11	48		56	47
12	0		3	34	12	27		56	48
12	37		3	29	12	51		56	55
13	7		3	26	13	18		56	52
13	34		3	11	13	43		56	58
14	8		2	50	14	15		57	16
14	30		2	44	14	42		57	27
14	54		2	36	15	6		57	34
15	23		2	20	15	33		57	52
15	45		2	17	15	58		57	46
16	16		2	39	16	23		57	24
16	37		2	34	16	43		57	46
16	56		2	23	17	2	269	57	47
17	15	90	2	34					
Feb. 10									
13	41	90	3	19	14	0	269	57	12
14	15		3	13	14	23		56	45
14	31		3	28	14	40		56	57
14	59		3	51	15	8		56	40
15	33		3	55	15	42		56	11
16	6		4	0	16	18		56	22
16	35		3	53	16	44		56	18
16	51		4	4	16	59		56	14
17	9		4	17	17	16		56	18
17	23	90	3	57	17	41	269	56	12
Feb. 11									
6	34	90	6	41	6	48	269	54	28
7	2		5	53	7	11		54	25
7	24		5	16	7	34		55	7
7	42		5	32	7	55		55	13
8	47		5	18	8	55		55	47
9	2		5	2	9	10		54	54
9	39		5	5	9	46		55	30
10	8		4	32	......			......	
10	24	90	4	20	10	33	269	56	0
Feb. 12									
6	26	90	5	35	6	39	269	55	24
6	54		5	45	7	3		54	50
7	20		5	56	7	35		55	30
7	56		5	42	8	9		55	7
8	25		5	35	9	0		56	14
9	14		3	44	9	31		56	6
9	21		4	0	......			......	
9	44		3	43	9	52		55	48
......			......		10	10		55	47
10	41		3	55	10	50		56	9
11	1		4	18	11	21		55	21
11	44		5	4	11	52		55	43
12	22		4	50	12	35		55	14
13	34		5	57	13	49		54	54
14	16		5	19	14	32		54	35
14	59		5	22	15	9		54	33
15	28		5	5	15	47		54	57
16	5		5	7	16	19		55	7
16	42		5	2	16	53		55	36
17	19		4	55	17	24		55	59
17	31	90	4	43	17	39		55	34
......			......		17	51	269	57	5

TABLE 2. METEOROLOGICAL DATA DURING THE OBSERVATIONS

JST				T_w	T_h	A_D	A_W	H	$e_{\max}$	$f_{\max}$	e_h	f_h
h	m	h	m	°C	°C	°C	°C	%	gr/m³	mb	gr/m³	mb
Feb. 5												
7	15 —	7	30	13.7	9.2							
7	33			13.7	9.7							
7	58 —	8	10	13.9	10.3							
8	31 —	8	43	13.5	11.2							
8	47 —	8	58	13.5	11.3							
9	4 —	9	17	13.7	11.3							
9	54				11.9	12.1	8.8	65	10.8	14.0	7.0	9.1
10	0 —	10	17	14.2	12.2	12.4	9.0	64	11.0	14.3	7.0	9.2
10	20 —	10	39	14.3	12.1	12.5	9.1	64	11.0	14.4	7.0	9.1
10	40 —	11	3	14.3	12.2	12.4	9.2	66	11.0	14.3	7.3	9.5
11	3 —	11	19	14.4	12.5	12.7	9.5	67	11.2	14.6	7.5	9.8
11	45 —	12	0	14.6	12.7	12.8	9.6	67	11.3	14.7	7.6	9.9
12	1 —	12	16	14.7	12.8	12.9	9.8	67	11.3	14.8	7.6	9.9
12	21 —	12	37	14.7	12.5	12.9	9.8	67	11.3	14.8	7.6	9.9
12	41 —	12	58	14.7	12.7	12.8	9.8	69	11.3	14.7	7.8	10.1
13	23 —	13	42	14.6	12.5	12.6	9.4	66	11.1	14.5	7.3	9.6
Feb. 6												
6	43 —	7	2	13.7	9.9	10.5	8.9					
7	3 —	7	26	13.8	10.0	11.2	9.3	78	10.2	13.2	8.0	10.3
7	33 —	7	43	13.8	11.9	11.7	9.6	79	10.5	13.7	8.3	10.8
7	57 —	8	8	13.7	12.2	12.2	9.8	74	10.8	14.1	8.0	10.5
8	9 —	8	21	13.9	12.0	12.0	9.7	75	10.7	14.0	8.0	10.5
8	23 —	8	34	13.9	12.9	12.4	10.1	75	11.0	14.3	8.2	10.6
8	41 —	8	51	13.9	12.2	12.7	10.2	73	11.1	14.6	8.1	10.7
8	53 —	9	5	13.9	12.3	12.4	10.2	76	11.0	14.3	8.4	10.9
9	11 —	9	22	14.0	12.2	12.5	10.0	73	11.0	14.4	8.0	10.5
9	23 —	9	34	14.0	12.7	13.1	10.4	71	11.4	15.0	8.1	10.6
9	46 —	9	59	14.0	12.7	13.1	10.5	72	11.4	15.0	8.2	10.8
10	1 —	10	12	14.0	12.5	12.8	10.2	72	11.2	14.7	8.1	10.6
10	14 —	10	25	14.0	12.7	12.9	10.3	72	11.3	14.8	8.1	10.7
10	32 —	10	43	14.1	12.7	13.2	10.5	71	11.5	15.1	8.2	10.7
10	45 —	10	55	14.2	12.9	13.4	10.6	71	11.7	15.3	8.3	10.9
11	0 —	11	12	14.2	14.5	13.7	10.8	70	12.0	15.6	8.4	10.9
11	20 —	11	31	14.2	13.6	13.8	10.8	70	12.0	15.8	8.4	11.1
11	32 —	11	44	14.2	14.5	14.0	10.9	69	12.2	16.0	8.4	11.1
11	55 —	12	5	14.2	14.0	14.4	11.2	68	12.4	16.4	8.4	11.2
12	25 —	12	36	14.2	14.4	13.8	11.0	71	12.0	15.8	8.5	11.2
12	40 —	12	51	14.3	16.3	15.0	11.7	68	12.9	17.1	8.8	11.6
13	0 —	13	11	14.4	16.0	14.7	11.5	68	12.7	16.7	8.6	11.4
13	13 —	13	24	14.4	15.8	14.2	11.5	72	12.3	16.2	8.9	11.6
13	28 —	13	39	14.4	15.7	14.7	12.1	74	12.7	16.7	9.4	12.3
13	41 —	13	51	14.4	15.0	15.4	12.5	72	13.2	17.5	9.5	12.6
14	1 —	14	12	14.3	15.2	15.0	12.7	77	12.9	17.1	9.9	13.2
14	18 —	14	32	14.3	15.3	15.0	12.8	78	12.9	17.1	10.1	13.4
14	37 —	14	49	14.2	15.3	15.0	12.9	79	12.9	17.1	10.2	13.5
14	53 —	15	5	14.3	15.3	15.0	13.0	80	12.9	17.1	10.4	13.7
15	11 —	15	24	14.2	15.0	15.3	13.3	80	13.2	17.4	10.6	13.9
15	29 —	15	41	14.2	15.0	15.4	13.0	76	13.2	17.5	10.1	13.3
15	47 —	15	59	14.1	15.0	15.2	12.8	76	13.1	17.3	10.0	13.1
16	3 —	16	15	14.1	14.9	14.8	13.1	83	12.8	16.9	10.6	14.0
16	19 —	16	30	14.1	14.7	15.0	13.4	84	12.9	17.1	10.8	14.4
16	33 —	16	43	……	14.7	15.1	13.1	80	12.9	17.3	10.3	13.8
16	52 —	17	3	14.2	14.6	15.2	13.2	80	13.1	17.4	10.5	13.9
17	11 —	17	24	14.2	14.3	14.8	13.0	82	12.8	16.9	10.5	13.8

TABLE 2. METEOROLOGICAL DATA DURING THE OBSERVATION (Continued)

J S T				T_w	T_h	A_D	A_W	H	e_{max}	f_{max}	e_h	f_h
h	m	h	m	°C	°C	°C	°C	%	gr/m³	mb	gr/m³	mb
Feb. 10												
13	25 —	13	50	15.0	13.1	13.5	5.0	20	11.8	15.4	2.4	3.1
14	0 —	14	15	14.4	13.3	13.4	5.8	27	11.7	15.3	3.2	4.1
14	35 —	14	55	14.4	12.7	12.8	5.5	28	11.2	14.7	3.1	4.1
15	30 —	15	44	14.3	11.8	11.9	5.2	31	10.6	13.9	3.3	4.3
16	10 —	16	20	14.2	11.8	11.8	5.3	33	10.6	13.8	3.5	4.5
16	46 —	17	3	14.2	10.7	10.6	4.3	32	9.7	12.8	3.1	4.1
17	12 —	17	23	13.4	10.0	10.2	4.0	32	9.5	12.4	3.0	4.0
Feb. 11												
6	28 —	6	50	11.8		4.4	0.6	46	6.6	8.3	3.0	3.8
7	40 —	8	0	12.2	4.5	5.2	1.6	50	6.9	8.8	3.5	4.4
8	50 —	9	0	12.4	4.3	4.6	1.6	57	6.6	8.5	3.8	4.8
9	15 —	9	38	12.1	3.9	4.0	1.6	65	6.4	8.1	4.2	5.3
10	15 —	10	35	13.4	3.7	3.8	2.4	78	6.3	8.0	4.9	6.2
11	30 —	11	40	13.2	3.0	4.0	2.5	78	6.4	8.1	5.0	6.3
Feb. 12												
6	28 —	6	44	10.4	3.5	4.6	1.2	51	6.6	8.5	3.4	4.3
7	6 —	7	15	13.2	4.9	5.3	2.0	54	6.9	8.9	3.7	4.8
7	27 —	7	37	13.6	6.8	6.9	2.6	45	7.7	9.9	3.5	4.5
7	44 —	7	55	13.4	7.5	7.5	3.0	44	8.1	10.4	3.6	4.6
8	8 —	8	19	13.6	8.4	7.6	3.3	47	8.1	10.5	3.8	4.9
8	24 —	8	44	13.6	8.3	9.2	5.2	53	8.9	11.7	4.7	6.2
8	45 —	8	55	13.8	9.7	8.3	3.6	43	8.4	11.0	5.1	4.7
9	42			14.1								
9	59 —	10	8		10.4	9.6	4.6	43	9.1	12.0	4.7	5.2
10	11 —	10	26	14.2	9.6	9.8	4.8	44	9.3	12.2	4.1	5.1
10	42 —	10	53	14.3	9.7	8.4	4.8	56	8.5	11.1	4.8	6.2
11	10 —	11	24	14.4	10.4	10.0	6.6	61	9.4	12.3	5.7	7.5
11	40 —	11	50	14.4	9.6	9.8	6.8	65	9.3	12.2	6.0	7.9
12	0 —	12	11	14.4	10.3	10.1	7.4	68	9.4	12.4	6.4	8.4
12	20 —	12	32	14.5	9.7	11.2	6.6	50	10.1	13.2	5.0	6.6
13	20 —	13	30	14.5	10.2	11.0	6.9	55	10.0	13.1	5.5	7.2
13	40 —	13	50	14.5	10.3	10.6	6.5	54	9.7	12.8	5.2	6.9
14	13 —	14	22	14.5	10.1	11.2	7.0	54	10.1	13.2	5.5	7.1
14	32 —	14	40	14.5	10.4	10.8	10.6	(98)	9.9	13.0		
15	2 —	15	9	14.6	10.0	10.9	6.4	51	10.0	13.1	5.1	6.7
15	20 —	15	28	14.6	10.4	11.0	6.2	47	10.0	13.1	4.7	6.2
15	41 —	15	50	14.7	10.1	10.8	6.2	49	9.9	13.0	4.9	6.4
16	1 —	16	10	14.8	10.0	10.4	6.6	57	9.6	12.6	5.5	7.2
16	15 —	16	21	14.8	10.4	10.4	6.9	60	9.6	12.6	5.8	7.6
16	25 —	16	33	14.9	10.0	10.2	6.0	52	9.5	12.4	4.9	6.5
16	36 —	16	43	14.9	10.0	9.8	6.1	57	9.3	12.2	5.3	7.0
16	47 —	16	54	14.9	9.0	9.0	5.7	61	8.8	11.5	5.4	7.0
17	6 —	17	15	14.9	7.6	7.7	5.1	67	8.2	10.5	5.5	7.0
17	21 —	17	28	14.9	7.4	7.5	4.8	66	8.0	10.4	5.3	6.9
17	32 —	17	40	14.8	7.2	6.2	4.8	81	8.0	10.4	6.5	8.4
17	46 —	17	56	14.8	6.9	6.4	4.5	75	7.4	9.4	5.6	7.0
18	16 —	18	19		7.1				7.5	9.6		
18	45			14.8								

apart from the theodolite.

c) The surface temperature of the sea water was measured outside of the shoal. The station was about 100 meters apart from the theodolite, and lay in the shadow of the islet in order to avoid the effect of solar radiation. Surface water was taken with a rubber sampling bucket, and its temperature was measured with a mercury-in-glass thermometer, except 5th (after 09^h 42^m) and 12th of February. On these two days, the temperature was measured with an electric thermister equipment set afloat on the same station. The results are given in the Table 2.

The surface temperature was measured also on a boat anchored about 2 miles away from the theodolite and lying on its line of sight to the objects. The results are given in the Table 3, the second column. On 5th of February, it was measured with a rubber sampling bucket and a mercury-in-glass thermometer, and on the 6th and 10th with the floating thermister equipment. On comparing the results with those in the Table 2, one can see the existence of effect of the land on water temperature near the shore.

d) The fourth and the fifth columns of the Table 2 gives the temperatures obtained from the dry and wet bulbs, respectively, of an Assmann aspiration psychrometer, which was set closely to the thermometer and at the same height as the telescope. The humidity

TABLE 3. WATER TEMPERATURE OBTAINED ON THE BOAT

J S T		T_W	J S T		T_W	J S T		T_W
h	m	°C	h	m	°C	h	m	°C
Feb.	5		Feb.	6		Feb.	10	
6	59	13.8	6	18	11.0	12	25	15.0
7	24	13.8	6	30	14.2	12	32	15.2
7	50	13.8	6	40	14.2	12	38	15.2
8	17	14.0	6	50	14.2	13	45	14.7
8	45	14.2	7	15	14.3	13	51	14.8
9	21	14.1	7	30	14.3	13	52	14.7
9	57	14.2	7	44	14.2	13	58	14.8
10	15	14.2	8	0	14.3	15	20	14.6
10	57	14.3	8	16	14.4	15	51	14.6
11	23	14.3	8	32	14.2	16	8	14.7
11	40	14.3	8	45	14.2	16	17	14.7
11	56	14.2	9	0	14.2	16	31	14.6
12	10	14.2	9	15	14.2	16	47	14.6
12	29	14.2	10	3	(14.3)	16	57	14.6
12	33	14.2	10	20	(14.2)	17	7	14.6
12	36	14.3	11	10	(14.6)	17	17	14.6
12	46	14.2	11	42	(14.6)	17	27	14.6
12	59	14.2	12	26	14.6	17	33	14.5
13	2	14.2	12	45	14.5	17	39	14.6
			13	0	14.6	17	43	14.6
			13	21	14.4	17	48	14.6
			13	42	14.3			
			13	53	14.5			
			14	2	14.2			
			14	13	14.2			
			14	22	14.2			
			15	42	14.7			
			15	45	14.6			
			16	0	14.6			
			16	15	14.5			
			16	31	14.6			
			16	45	14.5			
			17	0	14.4			
			17	15	14.5			
			17	30	14.4			
			17	40	14.4			
			17	45	14.6			
			17	54	14.4			
			18	3	14.5			
			18	11	14.5			

from these temperatures is shown in the sixth column of the Table.

e) All the thermometers and thermister employed at the present observations were carefully calibrated in Tokyo, before and after the observation session, and further before and after every day operation. The mean deviations of the thermometers from the standard one were smaller than $\pm 0°.2C$, and mean error of each measurement was also about $\pm 0°.2C$. The index error of the thermister equipment was less than $0°.1C$, and the mean error of its reading at field was about $\pm 0°.1C$, for air and water temperature measurements.

f) Height of the theodolite from the sea surface and its timely change were obtained through a tide staff observation; the mean height of the telescope was approximately 2.8 meters. The results are shown in the third column of the Table 4.

g) Weather conditions during the observing session are as follows:

Feb. 5: Cloudy at dawn, becoming fine at about 07^h00^m and thereafter with north-easterly wind of scale 3.

Feb. 6: Early in morning it was fine, but soon *cirrostratus* appeared with halo, and was clouded thickly after about 15^h00^m. Sea was calm.

Wind direction turned at about 13^h00^m from northeast to southwest and its scale was 1 to 2.

Feb 10: It was fine with fairly strong westerly wind of scale 4 through the afternoon.

Feb. 11: At dawn it had been clouded partly with little sunlight through *stratocumulus*. At about 07^h30^m it began to snow and changed to rain at about 10^h. Westerly wind of scale 3 turned to northeast.

Feb. 12: It was very fine all over the day. Wind of scale 1 to 2 turned at about noon its direction from west to northeast.

h) The mean values of the dips of horizon, from (L) and (R), have been tabulated in the second column of the Table 4. In this table, T_w, f_h and h are given through diagrams of their observed values against time.

It must be noted that the values of T_w have been taken from the Table 3. But those for Feb. 11 and 12 are estimated from a relationship, which has been obtained from the other day's data between the observed water temperatures at the shoal and on boat; since we could not carry out observation on boat on these two days.

The quantity f_w is the pressure of the water vapor for the temperature T_w, computed under and assumptions that the air just above the sea surface would be saturated by water vapor and that it might be of the same temperature as that of the water.

ΔT_w and Δf_w, which are tabulated in the Table 4, are defined as

$$\Delta T_w = T_h - T_w,$$

and

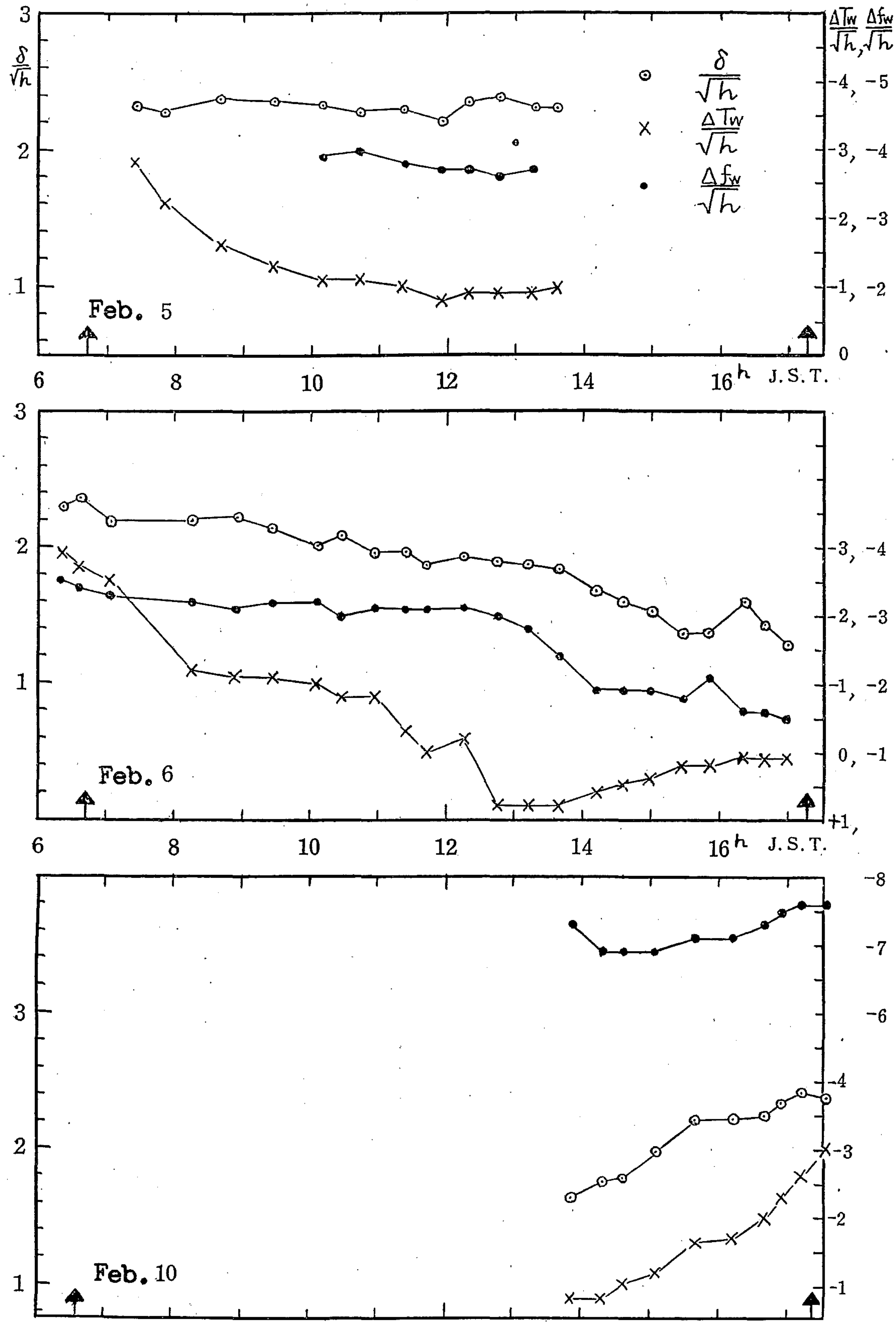
$$\Delta f_w = f_h - f_w, \tag{3}$$

TABLE 4. REDUCTIONS OF OBSERVATIONS

J S T	δ	h	$\frac{\delta}{\sqrt{h}}$	ΔJ	T_h	T_w	ΔT_w	$\frac{\Delta T_w}{\sqrt{h}}$	f_w	f_h	Δf_w	$\frac{\Delta f_w}{\sqrt{h}}$
h m	′	cm			°C	°C	°C		mb	mb	mb	
Feb. 5												
7 23	4.02	300	2.32	−0.30	<9.0	13.8	− 4.8	− 2.8	14.7			
7 50	3.93	303	2.27	−0.27	10.0	13.8	− 3.8	− 2.2	14.7			
8 40	4.13	306	2.36	−0.33	11.4	14.2	− 2.8	− 1.6	16.2			
9 26	4.12	307	2.35	−0.32	11.9	14.1	− 2.2	− 1.3	16.1			
10 9	4.08	306	2.33	−0.31	12.3	14.2	− 1.9	− 1.1	16.2	9.1	− 7.1	− 4.1
10 43	3.98	305	2.28	−0.28	12.4	14.3	− 1.9	− 1.1	16.4	9.4	− 7.0	− 4.0
11 22	3.98	302	2.29	−0.29	12.6	14.3	− 1.7	− 1.0	16.4	9.8	− 6.6	− 3.8
11 54	3.82	298	2.21	−0.24	12.8	14.2	− 1.4	− 0.8	16.2	9.9	− 6.3	− 3.7
12 18	4.03	294	2.35	−0.31	12.7	14.2	− 1.5	− 0.9	16.2	9.9	− 6.3	− 3.7
12 45	4.05	290	2.38	−0.32	12.7	14.2	− 1.5	− 0.9	16.2	10.0	− 6.2	− 3.6
13 14	3.90	286	2.31	−0.28	12.6	14.2	− 1.6	− 0.9	16.2	9.9	− 6.3	− 3.7
13 36	3.87	280	2.31	−0.28	12.6		− 1.6	− 1.0				
Feb. 6												
6 20	3.87	282	2.30	−0.27	(9.4)	14.2	− 4.8	− 2.9	16.2	10.4	− 5.8	− 3.5
6 37	3.99	288	2.35	−0.30	(9.6)	14.2	− 4.6	− 2.7	16.2	10.5	− 5.7	− 3.4
7 4	3.78	298	2.19	−0.23	9.9	14.2	− 4.3	− 2.5	16.2	10.5	− 5.7	− 3.3
8 16	3.89	315	2.19	−0.24	12.1	14.3	− 2.2	− 1.2	16.3	10.6	− 5.7	− 3.2
8 54	3.98	321	2.22	−0.26	12.3	14.2	− 1.9	− 1.1	16.2	10.7	− 5.5	− 3.1
9 27	3.88	325	2.15	−0.23	12.4	14.3	− 1.9	− 1.1	16.3	10.6	− 5.7	− 3.2
10 5	3.64	327	2.01	−0.16	12.5	14.3	− 1.8	− 1.0	16.3	10.7	− 5.6	− 3.1
10 28	3.78	326	2.09	−0.20	12.7	14.2	− 1.5	− 0.8	16.2	10.7	− 5.5	− 3.0
10 56	3.52	322	1.96	−0.13	12.9	14.4	− 1.5	− 0.8	16.4	10.9	− 5.5	− 3.1
11 25	3.53	317	1.98	−0.15	(14.0)	14.6	− 0.5	− 0.3	16.6	11.0	− 5.6	− 3.1
11 43	3.33	314	1.88	−0.09	14.5	14.6	0.0	0.0	16.6	11.1	− 5.5	− 3.1
12 14	3.38	306	1.93	−0.11	13.8	14.6	− 0.3	− 0.2	16.6	11.2	− 5.4	− 3.1
12 44	3.28	298	1.90	−0.09	15.8	14.5	+ 1.3	+ 0.8	16.5	11.4	− 5.1	− 3.0
13 13	3.18	291	1.87	−0.07	15.9	14.5	+ 1.4	+ 0.8	16.5	11.7	− 4.8	− 2.8
13 39	3.11	285	1.84	−0.06	15.7	14.4	+ 1.3	+ 0.8	16.4	12.3	− 4.1	− 2.4
14 12	2.78	276	1.67	−0.00	15.2	14.2	+ 1.0	+ 0.6	16.2	13.0	− 3.2	− 1.9
14 36	2.64	271	1.60	+0.03	15.3	14.4	+ 0.9	+ 0.5	16.4	13.3	− 3.1	− 1.9
15 0	2.52	267	1.54	+0.05	15.3	14.6	+ 0.7	+ 0.4	16.6	13.5	− 3.1	− 1.9
15 28	2.23	266	1.37	+0.10	15.0	14.6	+ 0.4	+ 0.2	16.6	13.7	− 2.9	− 1.8
15 52	2.26	267	1.38	+0.10	15.0	14.6	+ 0.4	+ 0.2	16.6	(13.2)	− 3.4	− 2.1
16 20	2.63	271	1.60	+0.03	14.8	14.6	+ 0.2	+ 0.1	16.6	13.9	− 2.7	− 1.6
16 40	2.40	276	1.44	+0.08	14.7	14.5	+ 0.2	+ 0.1	16.5	13.9	− 2.6	− 1.6
16 59	2.30	282	1.29	+0.11	14.6	14.4	+ 0.2	+ 0.1	16.4	13.9	− 2.5	− 1.5

TABLE 4. REDUCTIONS OF OBSERVATIONS (Continued)

J S T	δ	h	$\frac{\delta}{\sqrt{h}}$	ΔJ	T_h	T_w	ΔT_w	$\frac{\Delta T_w}{\sqrt{h}}$	f_w	f_h	Δf_w	$\frac{\Delta f_w}{\sqrt{h}}$
h m	′	cm			°C	°C	°C		mb	mb	mb	
Feb. 10												
13 51	3.06	349	1.64	+0.02	13.2	14.7	− 1.5	− 0.8	16.7	3.1	−13.6	− 7.3
14 19	3.23	338	1.76	−0.03	13.2	14.7	− 1.5	− 0.8	16.7	4.1	−12.6	− 6.9
14 36	3.26	332	1.79	−0.05	12.9	14.7	− 1.8	− 1.0	16.7	4.1	−12.6	− 6.9
15 4	3.59	321	1.97	−0.15	12.5	14.7	− 2.2	− 1.2	16.7	4.2	−12.5	− 6.9
15 38	3.87	308	2.21	−0.24	11.9	14.7	− 2.8	− 1.6	16.7	4.3	−12.4	− 7.1
16 12	3.82	296	2.22	−0.24	11.8	14.7	− 2.9	− 1.7	16.7	4.5	−12.2	− 7.1
16 40	3.79	285	2.25	−0.24	11.2	14.6	− 3.4	− 2.0	16.6	4.3	−12.3	− 7.3
16 55	3.92	280	2.34	−0.29	10.8	14.6	− 3.8	− 2.3	16.6	4.1	−12.5	− 7.5
17 13	3.99	274	2.41	−0.32	10.3	14.6	− 4.3	− 2.6	16.6	4.0	−12.6	− 7.6
17 32	3.88	269	2.37	−0.29	9.6	14.5	− 4.9	− 3.0	16.5	4.0	−12.5	− 7.6
Feb. 11												
6 44	6.11	273	3.70	−1.16	(4.8)	13.4	−(8.6)	− 5.2	15.3	3.8	−11.5	− 7.0
7 10	5.73	271	3.48	−0.99	(4.8)		−(8.6)	− 5.2		3.9	−11.4	− 6.9
7 32	5.08	270	3.09	−0.71	(4.7)		−(8.7)	− 5.3		4.1	−11.2	− 6.8
7 52	5.16	271	3.14	−0.75	4.6		− 8.8	− 5.3		4.3	−11.0	− 6.7
8 54	4.76	288	2.80	−0.53	4.4		− 9.0	− 5.3		4.7	−10.6	− 6.3
9 9	5.07	293	2.96	−0.69	4.3		− 9.1	− 5.3		4.9	−10.4	− 6.1
9 47	4.79	308	2.73	−0.56	4.0		− 9.4	− 5.4		5.4	− 9.9	− 5.6
10 11	4.27	319	2.39	−0.36	3.9		− 9.5	− 5.3		5.8	− 9.5	− 5.3
10 32	4.17	329	2.30	−0.32	3.8		− 9.6	− 5.3		6.2	− 9.1	− 5.0
Feb. 11												
6 33	5.09	289	2.99	−0.70	3.2	15.0	−11.8	− 6.9	17.1	4.2	−12.9	− 7.6
6 59	5.96	285	3.53	−1.08	4.5		−10.5	− 6.2		4.5	−12.6	− 7.5
7 28	5.22	283	3.10	−0.76	6.2		− 8.8	− 5.2		4.6	−12.5	− 7.4
8 4	5.29	283	3.14	−0.71	8.0		− 7.0	− 4.2		4.7	−12.4	− 7.4
8 25	5.17	286	3.06	−0.72	8.8		− 6.2	− 3.7		5.2	−11.9	− 7.0
9 7	3.75	296	2.18	−0.22	9.8		− 5.2	− 3.0		4.7	−12.4	− 7.2
9 26	3.97	303	2.51	−0.28	9.9		− 5.1	− 2.9		4.9	−12.2	− 7.0
9 48	3.96	310	2.25	−0.27	10.0		− 5.0	− 2.8		5.1	−12.0	− 6.8
10 10	4.22	319	2.36	−0.34	10.1		− 4.9	− 2.7		5.2	−11.9	− 6.7
10 46	3.88	334	2.12	−0.22	10.1		− 4.9	− 2.7		6.0	−11.1	− 6.1
11 11	4.48	346	2.41	−0.40	10.2		− 4.8	− 2.6		7.0	−10.1	− 5.4
11 48	4.68	358	2.47	−0.46	10.2		− 4.8	− 2.5		7.9	− 9.2	− 4.9
12 29	4.80	377	2.47	−0.49	10.3		− 4.7	− 2.4		7.0	−10.1	− 5.3
13 42	5.53	376	2.85	−0.78	10.4		− 4.6	− 2.4		7.0	−10.1	− 5.3
14 24	5.37	372	2.79	−0.72	10.4		− 4.6	− 2.4		7.1	−10.0	− 5.2
15 4	5.41	367	2.82	−0.74	10.3		− 4.7	− 2.5		6.7	−10.4	− 5.4
15 38	5.07	359	2.67	−0.61	10.3		− 4.7	− 2.5		6.3	−10.8	− 5.7
16 12	5.00	347	2.68	−0.59	10.2		− 4.8	− 2.6		7.3	−10.8	− 5.8
16 48	4.73	335	2.59	−0.51	9.1		− 5.9	− 3.2		7.0	−10.1	− 5.5
17 22	4.47	325	2.49	−0.42	7.5		− 7.5	− 4.2		6.9	−10.2	− 5.7
17 35	4.58	322	2.65	−0.46	7.3		− 7.8	− 4.4		7.6	−19.5	− 5.3
17 51	2.92	317	1.64	+0.02	7.0		− 8.0	− 4.5		7.0	−10.1	− 5.7


3
$\frac{\delta}{\sqrt{h}}$
2
1
Feb. 5
$\frac{\Delta T_w}{\sqrt{h}}, \frac{\Delta f_w}{\sqrt{h}}$
-4, -5
-3, -4
-2, -3
-1, -2
0
$\frac{\delta}{\sqrt{h}}$
$\frac{\Delta T_w}{\sqrt{h}}$
$\frac{\Delta f_w}{\sqrt{h}}$
6 8 10 12 14 16^h J.S.T.
Feb. 6
-3, -4
-2, -3
-1, -2
0, -1
+1,
Feb. 10
-8
-7
-6
-4
-3
-2
-1

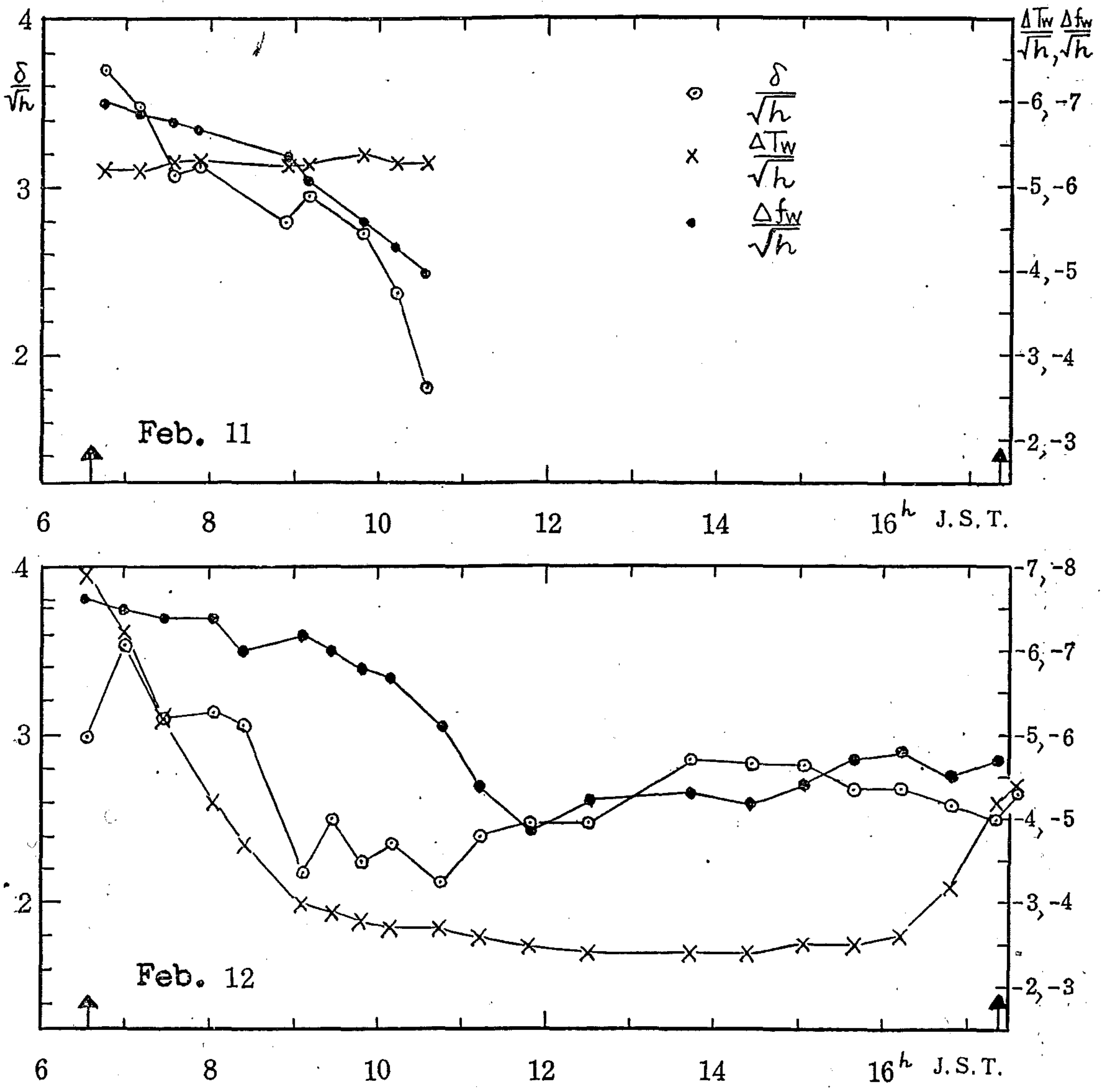


Fig. 1.

Diurnal changes of dip and respective differences of temperatures and humidities between air and sea surface. δ in min. of arc, T in deg. cent., f in mb., and h in meter. Times of sunrise and sunset are denoted by arrows.

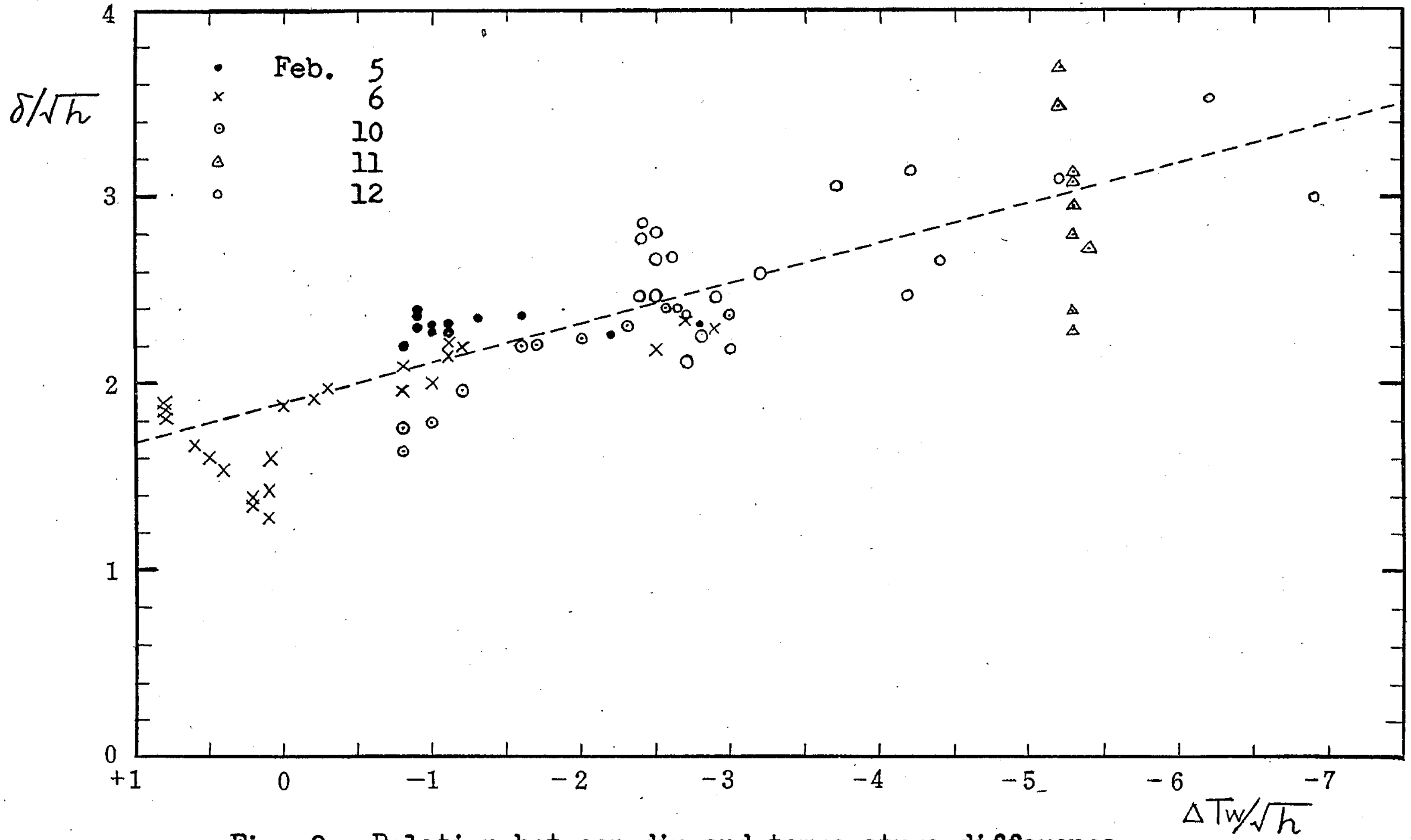


Fig. 2. Relation between dip and temperature difference measured with the same units as in Fig.1. Dashed line denotes the empirical relation (5).

respectively. These quantities should be brought into relations with dip, δ, under the classical idea.

Changes of $\delta/\sqrt{h}$, $\Delta T_w/\sqrt{h}$, and $\Delta f_w/\sqrt{h}$ with time are shown in the Fig. 1.

ΔJ, in the Table 4, stands for $T_h - T_o$, computed through the Freiesleben's formula (2):

$$\Delta J = -\left(\frac{\delta}{5.04}\right)^2 + 0.1123\,h \qquad (4)$$

3. Classical Empirical Formula.

We shall obtain the coefficients of the classical formula from our present observations. It should be remarked that our observations were generally made under the conditions that the water temperature was much higher than that of the air and mirages came out during the session. Therefore, it would be significant to see whether the dips could be represented in the form of (1) and, if so, what values the coefficients would take in these situations.

The observed $\delta/\sqrt{h}$ and $\Delta T_w/\sqrt{h}$ in the Table 4 are plotted in the Fig. 2.

The points lie fairly well on a straight line, except for the data obtained on Feb. 11, which scatter in almost parallel direction to the ordinate axis. As will be stated later, the dips on this day seem to be mainly influenced by water vapor in the air.

The empirical relation deduced from the entire data is obtained through the method of least squares:

$$\frac{\delta}{\sqrt{h}} = 1.845 - 0.221\frac{\Delta T_w}{\sqrt{h}} \qquad (5)$$
$$\pm 0.047 \pm 0.017 \qquad \text{(m. e.)},$$

and from the data except for those on Feb. 11,

$$\frac{\delta}{\sqrt{h}} = 1.832 - 0.233\frac{\Delta T_w}{\sqrt{h}} \qquad (5')$$
$$\pm 0.042 \pm 0.042 \qquad \text{(m. e.)},$$

We shall adopt the result (5) tentatively, since there is no significant difference between the two.

The constant term of (5) has rather larger values and the coefficient has also larger absolute values than those obtained by Akiyoshi(1936) and Freiesleben-Prüffer(1952), made in comparatively higher altitudes of observers. According to the results by the latter investigators, it seems to exist a tendency that the lower the height of an observer, the larger will become the absolute values of the coefficient. The present value is almost coincide with the result by Y. Tsukamoto obtained at 5 meter height (-0.23 ± 0.04, $\Delta T_w \sim 3°$ C). The constant term, which should be the standard value 1.776 by Bessel, has not so clear tendency to vary with height nor with temperature range. To

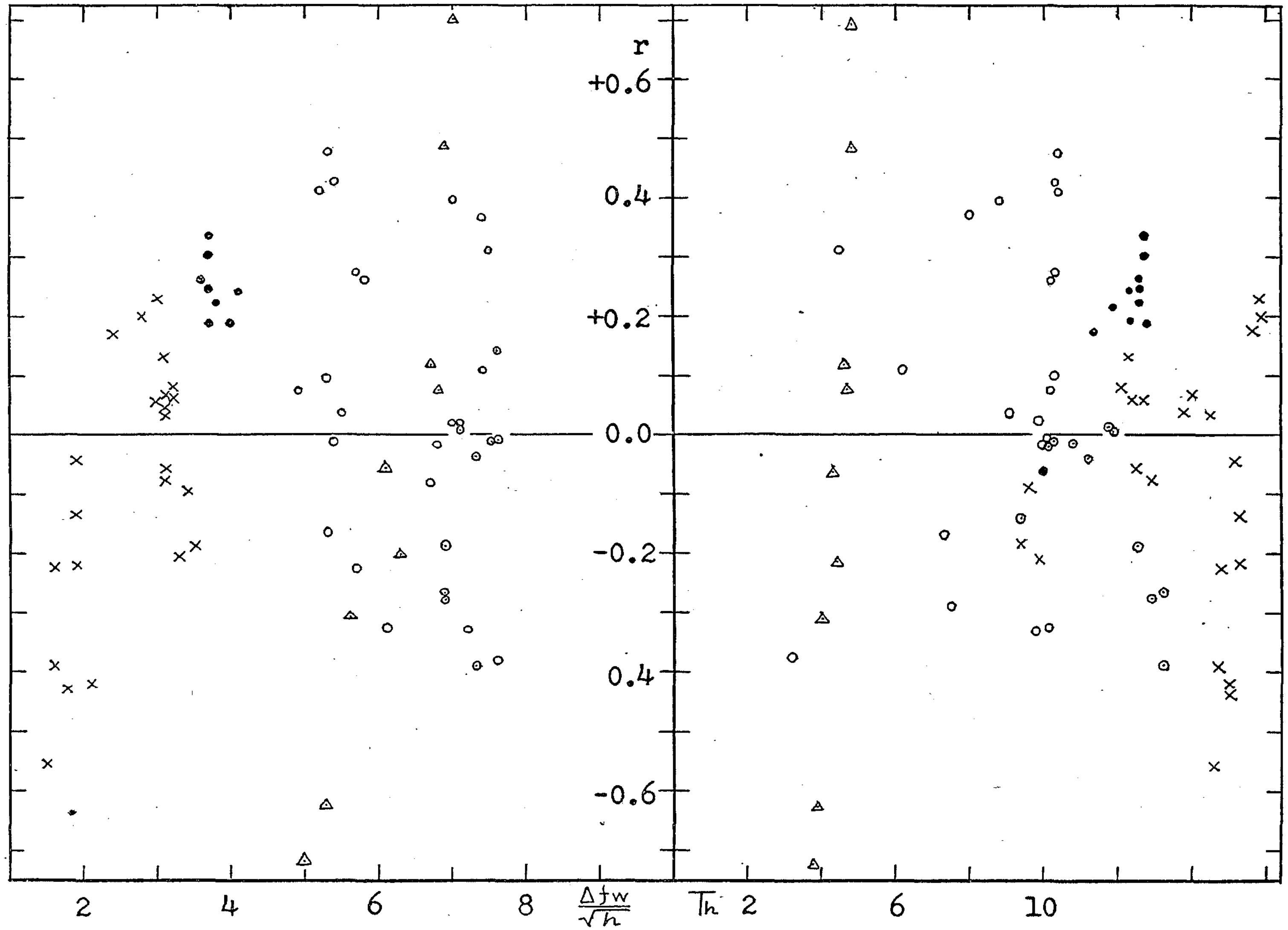


Fig. 3. Relations of residual of dip to humidity difference and to air temperature with the same units and symbols as in Fig.1.

make sure our conclusion, we have solved the least squares by imposing the value 1.776 upon the constant term and we have following results for the coefficient of $\Delta T_w/\sqrt{h}$ in the above two cases, respectively:

$$-0.239 \pm 0.031 \text{ (m. e.)},$$
$$-0.250 \pm 0.030 \text{ (m. e.)},$$

On considering the results we should remember that the observation were made near coast, not amid of ocean.

4. Dependence of Dip on Water Vapor.

In order to promote the accuracy of the empirical formula, relationships of dip with various meteorological elements have been investigated by several authors. T. Akiyoshi(1936) has exmained how the residuals, δ (obs.) $-\delta$ (comp.), have connections with atmospheric pressure and wind speed, but found so weak that they cannot affect the formula practically. Michler(1934) has concluded same results. On examining the residuals of $\delta/\sqrt{h}$ by (5) from the observed (Fig. 3), it is hard to find out their appreciable correlations with T_h and $\Delta f_w/\sqrt{h}$, and consequently it seems more reasonable to investigate the relationships in the following way.

In our present study we shall attempt to see how water vapor will affect on dip. The changes of $\delta/\sqrt{h}$ and $\Delta f_w/\sqrt{h}$ in the Fig. 1, particularly for Feb. 11, implies that there might certainly be a correlation between the two quatities. Naturally the existence of water vapor will reduce the refractive index of the air. Let the water vapor pressure be f and the atmospheric pressure p, then the refractive index of the mixture of the air and the vapor n is expressed by

$$n=n_1+(n_2-n_1)\frac{f}{p} \tag{6}$$

where n_1 and n_2 are the refractive indices of dry air and of water vapor, taking values, 1.000292 and 1.000256 at 0°C, 1 atm., respectively. Consequently, if more water vapor exists in the lower layers than in the upper, then dip is reduced, as if the temperature in the lower layers were higher than the one in the upper, and vice versa. Brocks(1951) has derived an extended formula of (2) taking the water vapor pressure into account.

The Fig. 4 shows the observed relation between the dips and $\Delta f_w/\sqrt{h}$, which is considerably close for the days Feb. 5, 6, and 11. But it should not be regarded as intrinsic but apparent, since the both quantities will be changed by a common quantity, probably temperature of air. If we assume that the relation would be true, we might have an empirical relation with our observed data,

$$\frac{\delta}{\sqrt{h}}=\underset{\pm 0.083}{0.886}-\underset{\pm 0.024}{0.067}\frac{\Delta T_w}{\sqrt{h}}-\underset{\pm 0.030}{0.358}\frac{\Delta f_w}{\sqrt{h}} \quad \text{(m. e.)},$$

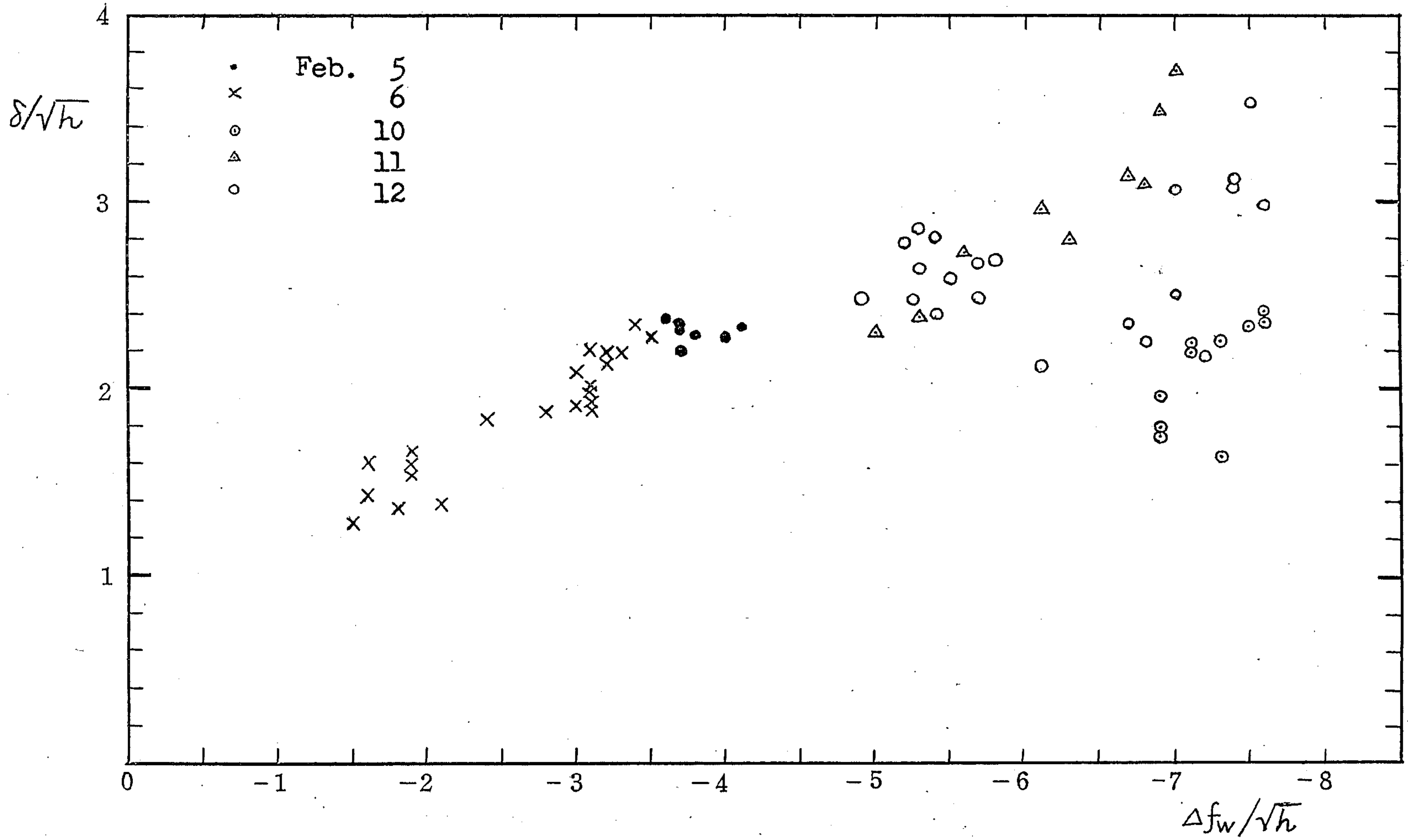


Fig. 4. Relation between dip and humidity difference measured with the same units as in Fig.1.

for Feb. 5 and 6, and

$$\frac{\delta}{\sqrt{h}}=\underset{\pm 0.056}{1.491}-\underset{\pm 0.014}{0.134}\frac{\Delta T_w}{\sqrt{h}}-\underset{\pm 0.014}{0.113}\frac{\Delta f_w}{\sqrt{h}} \quad \text{(m. e.),}$$

if all the data are included.

The effect of the water vapor will be divided into two parts: the one due to initially existed vapor and the other is the effect of evaporated vapor added to the initial amount. This must be more possible near land than amid of ocean. The latter vapor depends on the amount of solar radiation which will simultaneously increase the air temperature. Thus the vapor pressure pretends to depend on air temperature, and consequently change of dip with water vapor pressure may come out as shown in the Fig. 4. In fact, if we plot $\Delta f_w/\sqrt{h}$ against $\Delta T_w/\sqrt{h}$, then we find a weak but possible correlation between them (Fig. 5). Obviously, it appears that the data on Feb. 5 and 6 are likely to lie on a stright line, which is seen also in the Fig. 4. Therefore, it must be dangerous to deduce the dependence of dip directly from the Fig. 4.

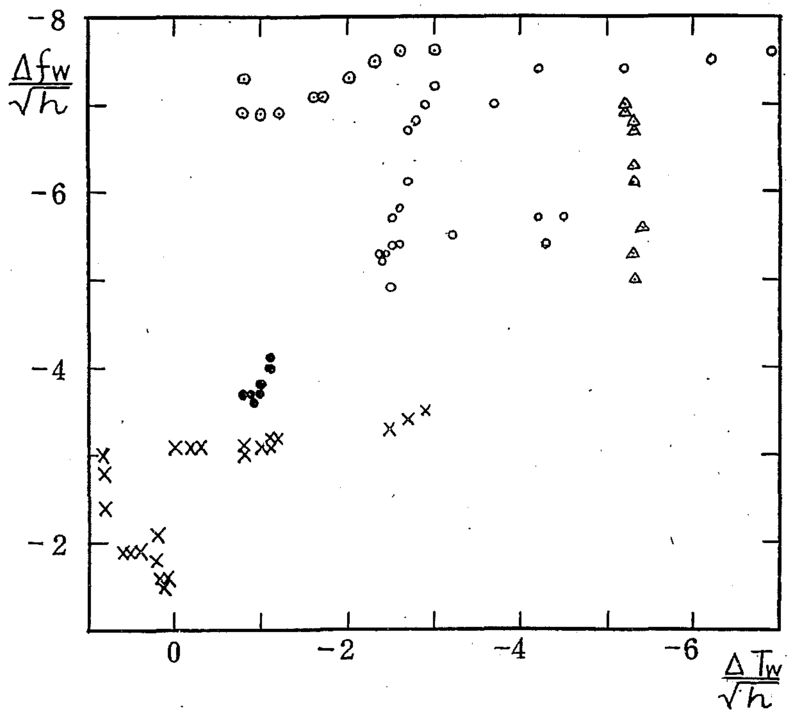


Fig. 5. Relation between temperature and humidity differences with the same units and symbols as in Fig. 1.

On looking at the data on Feb. 11, however, one can see that dips change rather closely with water vapor in spite of the constancy of $\Delta T_w/\sqrt{h}$ and h. Hence, we might regard that the change in the dips have caused by $\Delta f_w/\sqrt{h}$ alone, and as showing the true correlation of dip and water vapor. Though the accuracy of measurement of the pressure is not ample, we can derive the

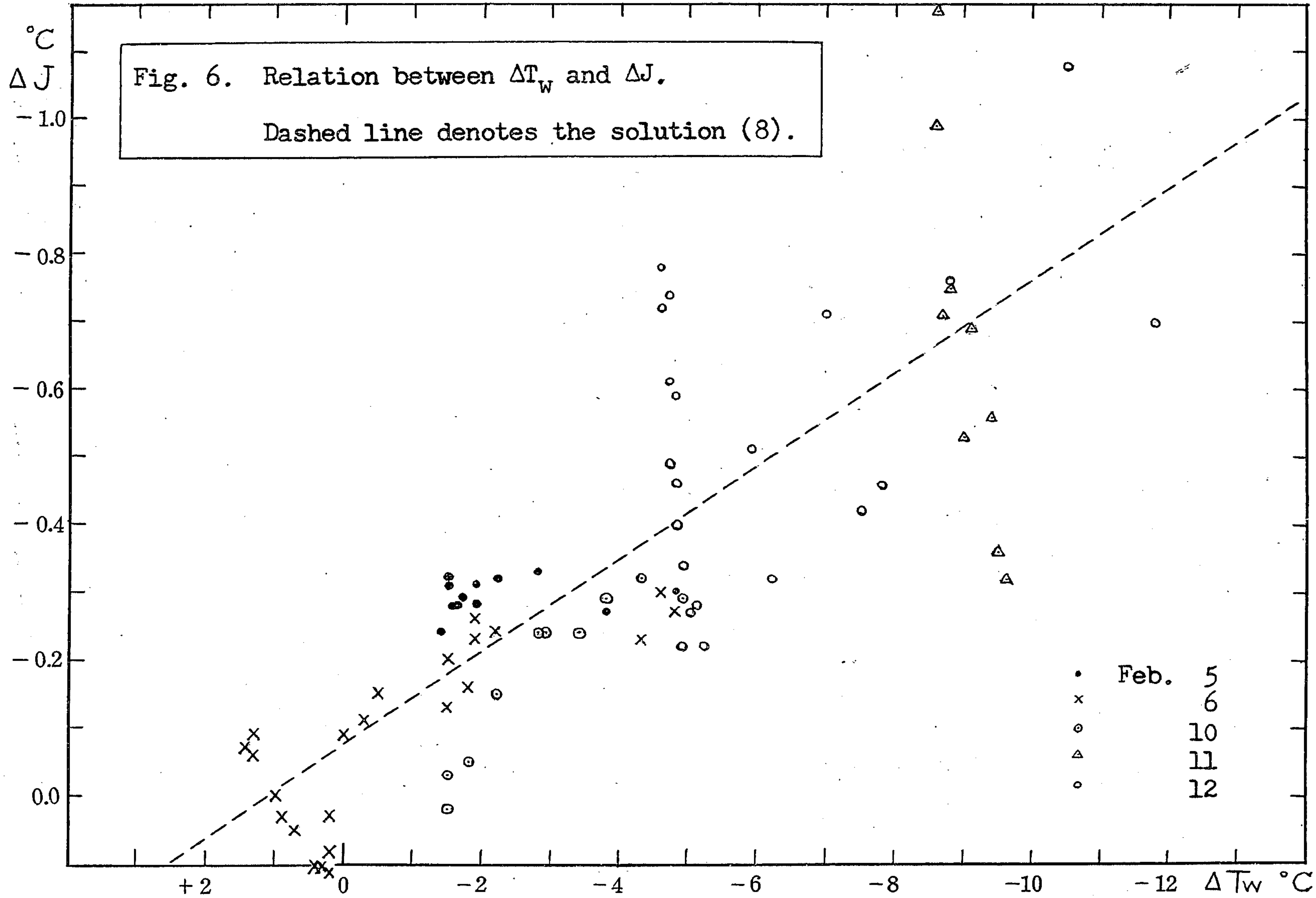


Fig. 6. Relation between ΔT_W and ΔJ. Dashed line denotes the solution (8).

following relation:

$$\frac{\delta}{\sqrt{h}}=-0.66-0.585\frac{\Delta f_w}{\sqrt{h}} \tag{7}$$
$$\pm0.53\pm0.086 \qquad \text{(m. e.)},$$

giving a large absolute value of the coefficient comparing to that in the previous case (5).

5. The Relationship between T_w and T_o-T_h.

T_o, the temperature of the air just above the sea surface, will not be naturally able to be expressed as a unique, simple function of T_h and T_w, since it should be determined by an initial temperature distribution in the atmospere and the water temperature and be dependent of time. Furthermore, it will be affected by distrubance of temperature field over the water by transportation of air of different temperature to the observed area.

However, dip has a fairly clear relation with $\Delta T_w/\sqrt{h}$, and consequently, it is inferred that there might be a simple relation between ΔT_w and ΔJ. We have plotted the two quantities in the Fig. 6; the points lie on a straight line with a correlation coefficient 0.80 ± 0.03, proving to be a strong correlation. They are related by

$$\Delta J=-0.075+0.0685\,\Delta T_w \tag{8}$$
$$\pm0.030\pm0.0061 \qquad \text{(m. e.)},$$

The interpretation of this fact seems difficult, as it is contradictory to the general concept of the nature of T_o mentioned above. Though it will be fully discussed in future, we shall give here possible interpretations:

a) The thermal situations in the lowest layers of the atmosphere would be in a stationary state.

b) The relation would not be real but apparent. Since the observation heights were not very large, T_h is very close to T_o, and ΔJ has the order of magnitude less than 1°.0C, which is only a few times of the mean error of the temperature measurement.

c) The equation (2) will yield enough accurate values of δ, but would not be suitable to get T_o inversely from given values of dip.

If we assume that the above relation to hold, dip will be expressed as

$$\delta=5.'04\sqrt{0.1123\,h-0.0685\,\Delta T_w+0.075}. \tag{9}$$

By investigating the accuracy of the equation, we have found that the new equation can yield the values of dip as closely to the observed ones as by the classical equation. Therefore, if physical ground of the relation (8) could be confirmed by further observational and theoretical investigations, it might be recommended to employ the equation (9) for general use, because of its physical propriety.

6. Conclusions.

Dip of sea horizon was observed with a theodolite at a height about 3 meters from the sea surface under conditions that the atmospheric temperatures were generally much lower than those of the surface water of sea, and that abnormal refraction, or mirage, consequently took place.

(1) It is found that the linear relation of dip to ΔT_w still holds and is expressed by the equation (5) in those conditions. The coefficient of $\Delta T_w/\sqrt{h}$ has rather a larger absolute value than those obtained from studies made with higher altitude of observers. If we impose the standard value 1.776 on the constant term, its absolute value becomes much larger.

(2) Dependence of dip on water vapor pressure is examined, but a definite relation is not concluded, owing to difficulty in accurate observations for humidity. However, if we might derive the relation between the vapor pressure and dip from our present observations, we would have the empirical relation (7).

(3) A rather close correlation between $T_h - T_o$ and $T_h - T_w$ is found out as expressed by the formula (8). It might be interpreted by an assumption that the thermal condition in the lowest parts of the atmosphere just above the sea surface would possibly be in a stationary state; but it will be needed to study temperature distribution and heat transfer in this region both observationally and theoretically, in order to interprete the relationship.

A new formula for dip

$$\delta = 5.'04\sqrt{0.1123 - \alpha\Delta T_w + \beta}$$

is suggested.

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